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Bayesian Inference for Seasonal ARMA Models: A Gibbs Sampling Approach

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Title of the Thesis: Bayesian Inference for Seasonal ARMA Models: A Gibbs Sampling Approach

Summary: The main objective of this study is to develop a Bayesian inference for multiplicative seasonal ARMA models by implementing a fast, easy and accurate Gibbs sampling algorithm. Bayesian analysis of seasonal ARMA model is difficult since the likelihood function is analytically intractable, which causes problems in prior specification and posterior analysis. Different solutions including Markov Chain Monte Carlo (MCMC) methods have been suggested in the literature for the Bayesian time series analysis. Bayesian time series analysis has been advanced by the emergence of MCMC methods especially Gibbs sampling method. The proposed algorithm does not involve any Metropolis-Hastings generation but is generated from multivariate normal and inverse gamma distributions. In addition, it is unconditional on the initial values, that is it assumes that the series starts at time t = 1 with unknown initial observations and errors. Moreover, it can easily test the significance of the interaction parameters which may be complicated to check in the classical framework. The proposed algorithm is illustrated using simulated examples and real data sets, its convergence is achieved and its accuracy is evaluated.

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Abstract

The main objective of this study is to develop a Bayesian inference for multiplicative seasonal ARMA models by implementing a fast, easy and accurate Gibbs sampling algorithm. Bayesian analysis of seasonal ARMA model is difficult since the likelihood function is analytically intractable, which causes problems in prior specification and posterior analysis. Different solutions including Markov Chain Monte Carlo (MCMC) methods have been suggested in the literature for the Bayesian time series analysis. Bayesian time series analysis has been advanced by the emergence of MCMC methods especially Gibbs sampling method. The proposed algorithm does not involve any Metropolis-Hastings generation but is generated from multivariate normal and inverse gamma distributions. In addition, it is unconditional on the initial values, that is it assumes that the series starts at time t = 1 with unknown initial observations and errors. Moreover, it can easily test the significance of the interaction parameters which may be complicated to check in the classical framework. The proposed algorithm is illustrated using simulated examples and real data sets, its convergence is achieved and its accuracy is evaluated.

Key Words: Bayesian time series analysis - Multiplicative Seasonal ARMA models - Prior distribution - Posterior distribution - Markov Chain Monte Carlo (MCMC) - Gibbs sampling.

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ABSTRACT

Time series models have been successfully applied in a great number of fields including economic forecasting, setting monetary and fiscal policies, state and local budgeting and financial management. Nonseasonal and seasonal ARMA models are a cornerstone of time series analysis and are commonly used in applications. However their Bayesian analysis is difficult since the likelihood function is analytically intractable, which causes problems in prior specification and posterior analysis.

Different solutions including Markov Chain Monte Carlo (MCMC) methods have been suggested in the literature for the Bayesian time series analysis. Bayesian time series analysis has been advanced by the emergence of MCMC methods especially Gibbs sampling method. The current study is interested in Bayesian inference of multiplicative seasonal ARMA models by implementing Gibbs sampling algorithm.

Firstly, the study defines the multiplicative SARMA model and introduces some special cases such as SAR, SMA, pure SAR, pure SMA and non-seasonal ARMA models. In addition, it introduces the principal Bayesian tools and concepts used in the thesis namely, the likelihood function of the multiplicative SARMA model, prior and posterior distributions.

After that, the study tries to obtain the conditional posterior distributions of the unknown parameters to be able to employ Gibbs sampling algorithm for estimating these parameters. It used a normal inverse gamma function as a prior distribution, which is a conjugate class, and used Jeffreys' prior, which is a special case of the normal inverse gamma class, when there is little information about the parameters a priori. All the conditional posterior distributions of the parameters are obtained and they are multivariate normal and inverse gamma distributions for which sampling techniques exist.

The proposed methodology for multiplicative SARMA model is unconditional on the initial values. In addition, various features of seasonal ARMA models, which may be complicated to check in the classical framework, may be routinely tested in the sampling based Bayesian framework. As an example, it can easily test the

significance of the interaction parameters which are the product of the nonseasonal and seasonal coefficients in the model.

The study evaluates the efficiency of the proposed methodology through four simulated examples. First a time series is generated from a given model with known parameters and then computed the Bayesian estimates via Gibbs sampling method. Finally we use Gibbs sampling results and efficiency criterion to evaluate the efficiency of the Bayesian estimates given the true parameters. In addition, the proposed methodology is used to analyze two real data sets: Federal Reserve Board Production Index and air-carrier freight data.

The main conclusion of the current study is the proposed methodology is accurate with no convergence problem and has some advantages over other methods in the literature. The proposed algorithm does not need sophisticated software and can be implemented by Excel for example.

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