



# **Newtonian and Non-Newtonian Fluids Flow through Porous Medium with Heat and Mass Transfer**

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submitted for the award of the Ph.D. degree  
in Applied Mathematics

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# SUMMARY

## **SUMMARY**

The aim of this thesis is to investigate some problems of Newtonian and Non-Newtonian fluids flow through porous medium with Heat and Mass Transfer. These problems are of great importance because of their multiple applications in various scientific fields, including biological, chemical, physical, medical and industrial such as materials manufactured by polymer extrusion, glass fiber and paper production, crystal growing, hot rolling and wire drawing annealing and tinning of copper wires, stretching of plastic film, cooling of metallic sheets or electronic chips, artificial fibers. The thesis consists of six chapters, with an Arabic and English summaries with a list of references

In chapter one, Introduction, we presented an introduction to the following topics:

- 1- Newtonian and non-Newtonian fluids.
- 2- The equations of flow for incompressible fluids.
- 3- Heat transfer and Mass transfer.
- 4- Dimensionless numbers in convective heat and mass transfer.
- 5- Flow in a porous medium.
- 6- Stretching surfaces.
- 7- Boundary layer theory.
- 8- Magnetohydrodynamics (MHD).
- 9- Micropolar fluid theory.
- 10- Nano fluids.

In chapter two, we investigated two problems elaborated in parts one and two as follows:

### **Part 1**

We studied the unsteady flow of MHD viscous fluid with heat and mass transfer through porous medium bounded by an infinite vertical plate. The governing partial differential equations, which describe the motion of the fluid, are converted dimensionless formulas by using similarity transformation method and solved analytically by using Laplace transform.

We obtained the formulas of velocity, temperature and concentration of the fluid as well as the skin friction and the rate of heat transfer. The effects of the physical parameters (chemical reaction, magnetic field, heat source, permeability, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number) of the problem on the velocity, temperature and concentration distributions are discussed numerically and illustrated graphically through a set of figures.

The results of this problem have been published in the *International journal of energy and technology*, vol. 3, pp. 1-11, 2011.

## **Part 2**

We studied the unsteady motion of MHD incompressible non-Newtonian fluid, which obeys the Powell-Eyring model with heat and mass transfer through porous medium near a moving vertical plate. The governing partial differential equations, which describe the motion of the fluid, are converted into dimensionless formulas by using similarity transformation method and solved analytically by using Laplace transform.

We obtained the distributions of the velocity, temperature and concentration of the fluid as well as the skin friction and the rate of heat transfer. The effects of the physical parameters (non-Newtonian material constant, chemical reaction, magnetic field, heat source, permeability, thermal Grashof number, mass Grashof number, Prandtl number, Schmidt number and Reynolds number) of the problem on these analytical solutions are discussed numerically and illustrated graphically through a set of figures.

The results of this problem have been published in the *International Journal of Applied Mathematics and Physics*, vol. 3, pp. 259-273, 2011.

In chapter three, we studied effect of the thermal radiation on flow and heat transfer over an unsteady stretching surface in a micropolar fluid with variable heat flux. The governing partial differential boundary layer equations are transformed into a system of ordinary differential equations. These equations are



solved numerically by mathematica program (Like shooting method). Comparison of the special cases of the numerical results is made with previous published results and we found that the results are good agreement. The effects of the unsteadiness parameter  $A$ , material parameter  $K$ , radiation parameter  $R$  and Prandtl number  $Pr$  on the flow and heat transfers are studied and shown graphically.

The results of this problem have been published in the *International journal of heat and technology*, vol. 30, pp. 93-98, 2012.

In chapter four, we studied the effects of thermal radiation and magnetic field on heat transfer in a micropolar fluid with variable viscosity along a vertical porous stretching surface in the presence of internal heat generation. The governing equations are transformed into a system of ordinary differential equations and solved numerically using Mathematical program. The special cases of the obtained results are checked against previous published work in order to access the accuracy of the numerical method and we found that the excellent agreement between them. Effects of the various parameters of the problem on the velocity, temperature and rate of the heat transfer are also displayed graphically and in a tabulated form.

The results of this problem have been published in the *International Research Journal of Engineering Science, Technology and Innovation (IRJESTI)*, vol. 2, pp. 8-16, 2013

In chapter five, we studied the steady flow over a nonlinearly stretching surface in a water-based Nano fluid containing different types of nanoparticles: Cu, Ag and  $\text{Al}_2\text{O}_3$ . The governing equations are transformed into a system of ordinary differential equations and solved numerically by using Mathematica program. The special cases of the obtained results are checked against previous published work in order to access the accuracy of the numerical method and we found that the in excellent agreement between them. Effects of the various parameters on the nanofluid velocity profiles, temperature profiles, skin friction and rate of heat transfer are also displayed graphically and in a tabulated form.

The results of this problem have been published in the *International Journal of Applied Mathematics*, vol. 28, pp. 1130-1139, 2013.

In chapter six, The effect of the magnetic field on the flow and heat transfer over an unsteady stretching surface embedded in a porous medium filled with nanofluid is studied. The governing equations are transformed into a system of ordinary differential equations and solved them numerically using Mathematica

program. The special cases of the obtained results are checked against previous published work in order to access the accuracy of the numerical method and we found the excellent agreement between them. Effects of the various parameters on the nanofluid velocity profiles, temperature profiles, skin friction and rate of heat transfer are also displayed graphically and in a tabulated form.

The results of this problem have been published in the *International journal of energy and technology*, vol. 5, pp. 1-8, 2013.

# CONTENTS

# Contents

Acknowledgements	
Summary	Page
Contents	
Chapter1: Introduction	
1.1 Newtonian and non-Newtonian fluids	1
1.1.1 Introduction	1
1.1.2 Newtonian fluids	1
1.1.3 Non-Newtonian fluids	3
1.1.4 Classification of non-Newtonian fluids:	4
1.1.4.1 <i>Time independent fluid behavior</i>	5
1.1.4.2 Time dependent fluid behavior	9
1.1.5 Some different models	13
1.2. The equations of flow for incompressible fluids	25
1.2.1 Equation of continuity	25
1.2.2 Navier-Stokes equations	25
1.2.3 Energy equation	27
1.2.4 Concentration equation	27
1.3 Heat transfer	28
1.3.1 Conduction	28
1.3.2 Convection	29
1.3.2.1 <i>Natural convection</i>	29
1.3.2.2 <i>Forced convection</i>	29
1.3.2.3 <i>Mixed convection</i>	30
1.3.3 Radiation	31
1.4 Mass transfer	32
1.4.1 mass transfer by diffusion	32
1.4.2 mass transfer by convection	33
1.4.3 mass transfer by change of phase	33
1.5 Dimensionless numbers in convective heat and mass transfer	33
1.5.1 <i>Reynold number</i>	33
1.5.2 <i>Grashof number</i>	34
1.5.3 <i>Prandtl number</i>	34
1.5.4 <i>Schmidt number</i>	34
1.5.5 <i>Skin friction</i>	35
1.5.6 <i>Nusselt number</i>	35
1.5.7 <i>Sherwood number</i>	35
1.5.8 <i>Mixed convection parameter</i>	36
1.6 Flow in a porous medium	36
1.6.1. Porous medium	39
1.6.2 Properties of Porous Media	39

1.6.3. <i>Application of porous medium</i>	39
1.6.3.1 <i>Chemical applications</i>	39
1.6.3.2 <i>Geological applications</i>	39
1.6.3.3 <i>Petroleum applications</i>	39
1.6.3.4 <i>Environmental applications</i>	39
1.7 Boundary layer theory	40
1.8 Stretching surface	42
1.9 Magnetohydrodynamics (MHD)	42
1.10. Micropolar fluid theory	44
1.11 <i>Nano fluids</i>	45
1.11 .1 Advantages of nanofluids	46
1.11 .2 Applications of nanofluids	46
1.12. <i>Survey on some previous studies related to this work:</i>	46
<b>Chapter2: Part1 Unsteady Motion of MHD Viscous Incompressible Fluid with Heat and Mass Transfer through Porous Medium near a Moving Vertical Plate</b>	
2.1.1 Introduction	51
2.1.2 Formulation of the problem	52
2.1.3. Solution of the problem	55
2.1.4. Results and discussions	61
2.1.5 Conclusion	77
<b>Chapter2.Part2: Effects of Heat and Mass Transfer on Unsteady MHD Free Convection Flow of Non –Newtonian Powell- Eyring Model through a Porous Medium Near a Moving Vertical Plate in Presence of Chemical Reaction and Heat Generation</b>	
2.2.1 Introduction	78
2.2.2 Formulation of the problem	78
2.2.3. Solution of the problem	82
2.2.4. Results and discussions	89
2.2.5 Conclusion	104
<b>Chapter3: Effects of Thermal Radiation and Magnetic Field on Heat Transfer in a Micropolar Fluid along a Vertical Stretching Surface with a Variable Viscosity and Internal Heat Generation</b>	
3.1 Introduction	105
3.2 Formulation of the problem	105
3.3 Numerical Solutions	113
3.4. Results and discussions	114
3.5 Conclusion	125
<b>Chapter4: Effect of Thermal Radiation on Heat Transfer over an Unsteady Stretching Surface in a Micropolar Fluid with Variable Heat Flux</b>	

4.1 Introduction	127
4.2 Formulation of the problem	127
4.3 Numerical Solutions	131
4.4. Results and discussions	135
4.5 Conclusion	140
<b>Chapter5: Effects of Magnetic Field and Heat Generation on Viscous Flow and Heat Transfer over a Nonlinearly Stretching Surface in a Nanofluid</b>	
5.1 Introduction	141
5.2 Formulation of the problem	141
5.3 Numerical Solutions	146
5.4. Results and discussions	147
5.5 Conclusion	157
<b>Chapter6: Effect of Magnetic Field on Flow and Heat Transfer over Unsteady Stretching Surface Embedded in a Porous Medium Filled with Nanofluid</b>	
6.1 Introduction	158
6.2 Formulation of the problem	158
6.3 Numerical Solutions	163
6.4. Results and discussions	166
6.5 Conclusion	177
<b>References</b>	179
<b>Arabic summary</b>	