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MAGNETO-ELECTRO GRAVITATIONAL OSCILLATION OF RADIATING FLUIDS.

**Thesis Submitted for the Degree of
Doctor of Philosophy (Ph.D.)
(Applied Mathematics)**

By

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Abstract

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The thesis is mainly concerned with some important stability problems which have important and crucial applications in several domains of science (See Chandrasekhar (1981), Kendal (1986) and Radwan (2005)).

In Chapter I, is survey contains the concept of stability and the analysis in terms of normal mode technique, Also we have introduced the subject of magnetohydrodynamic (MHD) and explained the basic (MHD) equations of motion. We also did write about some previous studies relating to the work in this thesis .

Chapter II, is devoted to studying the stability of streaming gas cylinder of radius R_0 surrounded by liquid of radius $q R_0$ ($1 < q < \infty$). This gas is penetrated by oblique magnetic field under the inertia , pressure gradient and electromagnetic force while the liquid is pervaded by a uniform axial magnetic field. The problem is formulated, solved and upon appropriate boundary conditions, the eigenvalue relation is derived and discussed analytically and the results are confirmed numerically.

Some results of the present (Chapter II) work has been already published in J. Applied Science Research 2(3) (2008) 241-247.

Chapter III, we have studied the stability of Axisymmetric fluid cylinder surrounding by medium .The model is acting upon the pressure gradient force, the self gravitating force, the electric force . The problem is formulated, solved and upon appropriate boundary conditions, the eigenvalue relation is derived and discussed analytically and the results are confirmed numerically to identify the stable and unstable domain .

Some results of the present (Chapter III) work has been already published in Australian Journal of Basic and Applied Sciences 4(3) (2008) 501-509.

Chapter IV, we have studied Electrohydrodynamic Stability of Self-Gravitating Oscillating Streaming Jets. The model is acting upon the pressure gradient force, the self gravitating force, the electric force. The problem is formulated, solved and upon appropriate boundary conditions, the eigenvalue relation is derived and discussed analytically and the results are confirmed numerically to identify the stable and unstable domain.

Chapter V, we have discussed the hydromagnetic self gravitating oscillation of radiating fluid with variable streams. The problem is formulated and the basic equations are constructed and solved using normal mode analysis. These results reveal that:

- (i) The self gravitating force is stabilizing according to Jeans criterion
- (ii) The electromagnetic force has strong stabilizing influence
- (iii) In the absence of radiation, the streaming whether it is uniform or non-uniform in the presence of other forces is stabilizing.
- (iv) In the presence of radiation as the streaming is variable function of coordinate, it is strongly destabilizing

Summary

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CHAPTER I

INTRODUCTION

I.1. The Concept Of Stability

The concept of stability is given by Chandrasekhar (1981). For the phenomena which occur in nature, it has to satisfy one more condition, namely that it must be stable to small disturbances. A smooth ball resting on the surface of a hemisphere is stable if the surface is concave upward, but it is unstable to small displacement if the surface is convex upwards.

The Nobile prize winner (1986), Chandrasekhar (1981) gave the basis of the stability theory in the fluid flows . Suppose that we have a hydrodynamic system which, in accordance with the equations governing it, is in a stationary state, i.e in a state in which none of the variables describing it, is function of time. Let $x_1, x_2, \dots, x_k$ be a set of parameters which define the system. The parameters will include geometrical parameters such as dimensions of the system, parameters characterizing the forces which may be acting on system, such as pressure gradient, temperature gradient, magnetic field, rotation , ...etc .

Specifically, we ask : if the system is disturbed, will the disturbance gradually die down, or the disturbance growing in amplitude in such away that the system progressively departs from the initial state and never reverts to it ?

In the first case, we say that the system could be reverted to initial position with respect to a particular disturbance. In the second case it is said that the system is unstable and this is the case even if there is only one special mode of disturbance with respect to which it is unstable . The

system can't be considered stable unless it is stable with respect to every possible disturbance to which it can be subjected .

If all initial states are classified as stable or unstable , according to the criteria state, there in the space of the parameters $x_1, x_2, \dots, x_k$, the locus which separates the two classes of states defines the states of marginal stability of the system. By this definition, a marginal state is a state of neutral stability .

The locus of the marginal states in the $(x_1, x_2, \dots, x_k)$ space will be defined by an equation of the form

$$\sum (x_1, x_2, \dots, x_k) = 0 \quad (1.1)$$

The determination of this locus is one of the prime objects of an investigation on hydrodynamic and magnetohydrodynamic stability .

In thinking of the stability of hydrodynamic system it is often convenient to suppose that all parameters of the system , save one, kept constant while the chosen one is continuously varied . We shall then pass from stable to unstable states when the particular parameter , which we are varying, takes a certain critical value. We may say that in stability arises at this value of the chosen parameter when all the others have there reassigned values .

States of marginal stability can be one of the two kinds. The two kinds correspond to the two ways in which the amplitude of a small disturbance can grow or be damped : they can grow (or be damped) periodically ; or they can grow (or be damped) by oscillations of increasing (or decreasing) amplitude . In the former case, the transition from stability to instability takes place via a marginal state exhibiting a stationary pattern of motions. In the later case, the transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency .

I.2. Stability Techniques

There are several techniques for solving the stability problems. We may write down here about some of them e.g. the method of multiple time scale, the variations principle method, the energy method and the most familiar method which is the normal mode analysis. However every method has its advantages and disadvantages. In the present work we have used the method of normal mode technique for the perturbations analysis cf. Chandrasekhar (1981). The mathematical treatment of a problem in stability theory, generally proceeds along the following lines. We start from initial flow which represents a stationary state of the system, then we let the flow suffer an infinitesimal disturbance and then we first obtain the equations governing these infinitesimal disturbances . A perturbation analysis depends on the fact that there is an equilibrium condition of the dependent variable which satisfies the equations of motion together with the boundary conditions. The linearized equations of motion are obtained by neglecting all products and higher orders of the perturbation variables . Each of the variables $Q_1(x_1, x_2, x_3, t)$ appearing in the equation of the motion is expanded about its equilibrium value $Q_0(x_1, x_2, x_3, t)$ in the following manner

$$Q_1(x_1, x_2, x_3, t) = Q_0(x_1, x_2, x_3, t) + \delta Q_1(x_1, x_2, x_3, t) + \delta^2 Q_2(x_1, x_2, x_3, t) + \dots \quad (1.2)$$

where δ is the amplitude of the perturbation .

From the use of normal mode analysis method we obtain the linearized form of the basic equations and apply Fourier analysis for an arbitrary perturbation .

The resulting boundary value problem is then solved , and the constants of integration are obtained by applying the boundary conditions. The criterion of stability is obtained from which one can deduce the regions of instability and those of stability upon discussing this criterion analytically . Verification of the obtained results may be done upon numerical studies via computer. The numerical data should be tabulated and presented graphically. One may go further step to compare the present results with experimental results if they are exist .

I.3. Magnetohydrodynamic Basic Equations

Here, we formally consider two groups of equations. The first group of equations is the hydrodynamic equations and the second group corresponding to Maxwell's electromagnetic field equations. There will be coupling of the two sets of equations, which appear in the boundary conditions .

In ordinary hydrodynamics of real fluid , the equations of motion of viscous fluid are given by Navier - Stockes' vector equation

$$\rho \left(\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \right) = \underline{F} - \nabla P - \lambda \nabla \wedge (\nabla \wedge \underline{u}) \quad (1.3)$$

where ρ , $\underline{u}$, P , and λ are the fluid mass density , velocity vector , kinetic pressure and fluid viscosity coefficient , and $\underline{F}$ is the external force per unit volume . $\underline{F}$ could be the self-gravitating , capillary , electromagnetic , electrodynamics force ,, etc .

If the fluid is inviscid , then equation (3) reduce to Euler's Equation

$$\rho \left(\frac{\partial \underline{u}}{\partial t} + (\underline{u} \cdot \nabla) \underline{u} \right) = \underline{F} - \nabla P \quad (1.4)$$

when the fluid is a heterogeneous and compressible , then we have the equation of mass conversation , which is

$$\frac{\partial \rho}{\partial t} + (\underline{u} \cdot \nabla) \rho = 0 \quad (1.5)$$

beside the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{u}) = 0 \quad (1.6)$$

for an incompressible fluid , equation (1.6) reduces to

$$\text{div } \underline{u} = 0 \quad (1.7)$$

for an irrotational motion , the velocity vector may be expressed as a gradient of a potential function , say Φ

Therefor, the velocity potential Φ

$$\underline{u} = -\text{grad } \Phi \quad (1.8)$$

consequently , for incompressible – irrotational flow , we have

$$\nabla^2 \Phi = 0 \quad (1.9)$$

Maxwell equations for a medium fluid under the influence of electromagnetic fields , can be written in the following form

$$\text{div } \underline{D} = q \quad (1.10)$$