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On The Behavior of Strategies in Repeated Games

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(Pure Mathematics)

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Table of Contents

List of Figures

List of Tables

List of Abbreviations

AD	Advantageous strategy
$AllC$	The strategy that cooperate in all round of game
$AllD$	The strategy that defect in all round of game
DIS	Distinguish strategy
ESS	Evolutionary stable strategy
IPD	Iterated prisoner’s dilemma game
NE	Nash equilibrium
PD	Prisoner’s dilemma game
RD	Risk-dominant strategy
TFT	Tit – For Tat strategy
$WSLS$	Win – Stay Lose – Shift strategy

Summary

The theory of iterated games provides a systematic framework to explore the players' relationship and analyzing the behavior of rational players in a long-term. In these games, the behavior of strategies, especially in the cooperative behavior, becomes a dilemma, this dilemma arises when two cooperators receive a higher payoff than two defectors. Since in the Prisoner's Dilemma game defectors dominate cooperators unless a mechanism for the evolution of cooperation is at work, thus, the purpose of this thesis is to study the cooperative behavior of strategies in the iterated prisoner’s dilemma game (IPD) and study how can this behavior evolve between players.

This thesis consists of four chapters, the first chapter is introductory and defines basic terminology used in the thesis. Many important topics about game theory have been addressed in this chapter, for example, the nature of games, describing strategic games, classifications of games, games in extensive and in normal form, mixed and pure strategies and the concept of a solution and Nash Equilibrium which play important role in the theory of games.

In chapter two, we provide a brief introduction about cooperation in the theory of games, the definition of cooperative dilemma and cooperation in repeated games. Moreover, we present the evolutionary game theory. This includes evolutionary stable strategies (ESS), evolutionary game dynamics, relationship between evolutionary and Nash equilibria and some examples on evolutionary stable strategies (ESS).

In the third chapter, we introduce some mechanisms for evolving the cooperative behavior in PD game. Also, we study the evolution of the cooperative behavior by combining two or three mechanisms together and find the transformed matrices for each situation. We derive the conditions for the evolution of cooperative behavior and study the (ESS) property of the strategies. Moreover, we derive the necessary conditions that make the cooperation risk- dominant and advantageous in a population in the context of the PD game. The results in this chapter have been published in *Journal of Game Theory* in 2015 entitled “Essam El-Seidy, Ali M. Almuntaser. (2015).

On The Evolution of Cooperative Behavior in Prisoner's Dilemma. Journal of Game Theory, 4(1): 1-5.".

In the last chapter, we study the IPD game in which there is a relationship ($0 \leq r \leq 1$) between the players . We assume that ,when the players play the IPD game there is some noise , i.e. in each round, a player makes a mistake with probability ε leading to the opposite move. We define the transition rule of each automaton that depends on the initial state of the game and on the payoff of the last move. Then, we describe the method that we shall follow to compute the $16\, x16$ payoff matrix for IPD game with noise played by finite state automata. After calculating the payoff matrix we study the effect of different values of average relatedness and different values for the payoff values $(R\ ,S\ ,T\ ,P)$ on the behavior of the 16 strategies. The findings in this chapter have been published in *International Journal of Scientific & Engineering Research* in 2015 under the name "Essam El-Seidy, Salah El Din S.Hussien and Ali M. Almuntaser (2015). On The Behavior of Strategies in Iterated Games Between Relatives , International Journal of Scientific & Engineering Research, Volume 6, Issue 4, April-2015.".

Chapter 1 Basic Concepts and Definitions of Game Theory

1.1 Introduction

It as often as possible happens that you are confronted with making a choice or choose a best strategy from a few decisions. For example, you may need to choose whether to put resources into stocks or bonds, or you may need to pick a hostile play to use in a football game. In both of these examples, the outcome relies on something you can't control. In the first example, your success partly depends on the future behavior of the economy. In the second example, it depends on the defensive strategy chosen by the opposing team. We can model situations like these using **game theory**.

Game theory is a study with the principle motivation behind discovering a response to the question: how to respond in both conflict and cooperation situations, as well as combined ones. This idea pre-characterizes a condition that there must be no less than two sides of a connection towards one another to discuss conflict/cooperation.

Game theory is a well -developed field of study has pulled in a portion of the world's most prominent mathematicians, won two Nobel Prizes and is even credited with winning the Cold War. The roots of game theory go far back in time. Late work recommends that the division of a legacy depicted in the Talmud (in the early years of the first thousand years) predicts the modern theory of cooperative games and, in 1713, James Waldegrave wrote out a strategy for a card game that provided the first known solution for a two player game.

Regardless of these early efforts, the book The Theory of Games and Economic Behavior by Von Neumann and Oskar Morgenstern (published in 1944) is typically credited as the beginning of the formal investigation of game theory. This spearheading work concentrated on finding unique strategies that allowed players to minimize their maximum losses (minimax solution) by considering, for every possible strategy of their own, all the possible responses of other players. Expanding upon von Neumann's prior work on two player games where the winnings of one player are equal and contrary to the losses of his opponent (zero-sum) and where each player knows the strategies available to all players and their consequences (perfect information), von Neumann and Morgenstern extended the minimax theorem to include games involving imperfect information and games with more than two players (see [63]).

The brilliant time of game theory happened in the 1950s and 1960s when scientists concentrated on finding sets of strategies, known as equilibria, to “solve” a game if all players behaved rationally. The most well known of these is the Nash Equilibrium proposed by John Nash, later made celebrated in the film "A Beautiful Mind" featuring Russell Crowe. A Nash equilibrium exists if no player can singularly move to enhance his own outcome. In other words, they have no impetus to change, since their strategies are all the better they can do given the activities of alternate players. Nash additionally made critical commitments to bargaining theory and examined cooperative games where dangers and guarantees are completely tying and enforceable. In 1965, Reinhard Selten introduced the concept of sub-game perfect equilibria, which describes strategies that deliver a Nash equilibrium across every sequential sub-game of the original game.

John Harsanyi formalized Nash’s work and developed incomplete information games in 1967. He, alongside John Nash and Reinhard Selten, won the Nobel Prize for Economics in 1994. Another vital commitment to game theory during the 1950s and 1960s was Luce and Raiffa's book, Games and Decisions. The Prisoner's Dilemma, presented by the RAND Partnership and exceptionally well known to any MBA understudy, is additionally a result of this period.

Further adding to the praise of game theory, another Nobel Prize was granted to game theorists, Robert Aumann and Thomas Schelling, in 2005. Schelling used game theory in his 1960 book The Strategy of Conflict to clarify why credible threats of nuclear annihilation from the U.S. and the former Soviet Union were counterbalancing through mutually assured destruction and therefore were not likely to be used. Aumann's work was mathematical and centered around whether cooperation expands if games are continually repeated rather than played out in a single encounter. He demonstrated that collaboration is more outlandish when there are numerous members, at the point when communications are occasional, when the time skyline is short or when others' activities can't be plainly watched.

These days game theory is identified with different areas, for example, ecology and biology, in particular related to evolution. In these areas, the individual's behavior does not rely on rationality, but on different perspectives, for example, fitness. Consistently, game theory has been connected to a wide range of fields of study, including artificial intelligence, bargaining, political science and real world business decisions (see [10-62]).

In this chapter, we will present a critical introduction to basic concepts of game theory. Likewise, we shall introduce the game-theoretic idea in least difficult terms. These include the nature of games, describing strategic games, classifications of games, games in extensive and in normal form, mixed and pure strategies, and the concept of a solution and Nash Equilibrium which play important role in the theory of games.

1.2 The Importance of Game Theory

Game theory is all over nowadays. After thrilling a whole generation of post-1970 economists, it is spreading like a bushfire through the sociologies. Two noticeable game theorists, Robert Aumann and Oliver Hart, clarify the fascination in the following way:

"Game Theory may be viewed as a sort of umbrella or 'unified field 'theory for the rational side of social science, it does not use different, ad hoc constructs, it develops methodologies that apply in principle to all interactive situations".

(Aumann and Hart, 1992)

Obviously, you may say, two professionals would say that, wouldn't they. Be that as it may, the perspective is broadly held, even among evidently unengaged gatherings. Jon Elster, for example, a surely understood social scholar with extremely various intrigues, comments in a comparative manner:

"If one accepts that interaction is the essence of social life, then game theory provides solid micro foundations for the study of social structure and social change".

(Elster, 1982)

The focal reason for game theory is to study the strategic relations between supposedly rational players. It accordingly investigates the social structures in which the results of a player's activity depend, consciously for the player, on the activities of alternate players. Game theory is for the most part separated into two branches, despite the fact that there are extensions that join the two. Non-cooperative game theory studies the equilibrium states that can result from the autonomous behavior of players unable to define irrevocable contracts. Cooperative game theory concentrates on the consequences of recreations administered by both individual and aggregate criteria of sanity, which may be forced by a specialist at some predominant level (see [29]).

Game theory's characteristic field of use is economic theory, the economic system is seen as a huge game between producers and consumers, who execute through the intermediation of the market. It can be all the more particularly connected to circumstances outside the domain of superbly aggressive markets, i.e. situations in which the agents get some control over the fixing of prices (imperfect competition, auction mechanisms, wage negotiations). It can be connected just as well to relations between the state and specialists, or to relations between two states. It can in this manner be viewed as a general grid for the sociologies and be connected to social relations as concentrated on in political science, military strategy, sociology, or even relations between animals in biology (see [10]).