



Cairo University

# **Further studies on punching problem: Theoretical basis and warping effects**

By

**Eng. Mohamed Rashwan Rabie Rashwan**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Structural Engineering

**FACULTY OF ENGINEERING, CAIRO UNIVERSITY**  
**GIZA, EGYPT**  
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**Title of Thesis:**

**Further studies on punching problem: Theoretical basis and  
warping effects**

**Key Words:** Boundary element method; punching parameters due to warping; ACI punching formula

**Summary:**

This thesis has two objectives:

The first objective is a pioneering step to furnish the theoretical basis behind the ACI 318 semi-empirical punching formula. Principles of the Boundary Element Method (BEM) and the theory of elasticity are adopted to establish the rationale behind the punching shear stress equation proposed in ACI 318. The present research further substantiates this effort through developing a BEM-based software for checking punching limit state for columns of any irregular cross-section shape – a task that ACI equation falls short of satisfying. The second objective is proposed to the ACI equation for checking punching a new term in order to include the effects of warping. A novel method for estimating the effects of warping of slab internal columns on punching stresses is introduced. In doing so, new punching parameters  $\gamma_{vw}$  and  $J_w$  are presented and evaluated/calibrated to include the effects of warping on punching and moment transfer analysis.

## DEDICATION

To my wonderful mother **Wedad Nabih ELGARHY** to whom, I owe too much. **Lord**

Forgive her, grant her mercy.

To my father **Rashwan Rabie Rashwan GOD** save him and give him health and strength.

To my brothers **Dr. Ahmed, Eng. Mahmoud** and **Islam**, who always supported me.

To my lovely wife **Hend** I am deeply thankful for her love, support, and sacrifices.

To my beautiful daughter **Aseel**, who stole my heart since day one.

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## NOTATIONS

$x$	The field points
$y$	Field point at the internal column domain
$u_j(x)$	boundary displacements
$t_j(x)$	boundary tractions
$ex_i, ey_i$	are the distances between the center of gravity of $i^{th}$ sub-column and the center of gravity of the column sub-division along x and y directions
$q_x, q_y$	vertical shear stresses, retrieved at each point from applying the boundary element method and the principles of the theory of elasticity
$I_x, I_y$	are second moments of area of the critical section
$U$	is the vertical displacement and rotations of the sub-columns
$A$	Column area
$E$	modulus of elasticity
$M_F$	is the total moment at the critical section due to shearing forces
$M_m$	is the total moment at the critical section due to bending and twisting moments.
$Y$	source point located at each sub column center
$A_c$	is the area of concrete of the assumed critical section
$J_w$	property of column
$c_w$	is the length between center of column to the corner of critical section
$F_{ck}$	is the forces in the desired sub columns due to boundary displacements
$K_{ck}$	the stiffness of sub columns
$T_{ij}(\xi, x)$	Traction component for fundamental solution between source point and boundary point.
$U_{ij}(\xi, x)$	displacement component for fundamental solution between source point and boundary point.
$C_{ij}(\xi)$	jump term; $C_{ij}(\xi) = \delta_{ij}/2$ if $\xi \in \Gamma$ (smooth boundary) and $C_{ij}(\xi) = \delta_{ij}$ if $\xi \in \Omega$
$M_{\alpha\beta}$	Bending moment
$Q_{3\alpha}$	Shear stresses
$\beta\gamma_{vw}$	Bi-moment portion transferred to the column by eccentric shear stress

$\Gamma$	boundary
$\delta(\xi, x)$	Dirac delta; $\delta(\xi, x) = \infty$ at $x=\xi$ , and $\delta(\xi, x) = 0$ elsewhere.
$\delta_{ij}$	identity matrix; $\delta_{ij} = 0$ when $i \neq j$ , and $\delta_{ij} = 1$ when $i=j$ .
$\lambda$	Shear Factor.
$\gamma_v$	Portion of the unbalanced moment transferring by shearing force
$\gamma_f$	Portion of the unbalanced moment transferring by bending and twisting moments
$\nu$	Poisson's ratio.
$\xi$	source point.
$\Omega$	domain
$\nabla^2$	The two dimensional Laplace operator
$n_\beta$	The direction cosines of the outward normal to the boundary.
$\omega$	sectorial coordinate
$\theta_x, \theta_y$	are the rotations of the column
$w$	Vertical displacement of the column
$\theta_z$	Warping displacement of the column
$\tau_u$	punching shear stress

## ABSTRACT

This thesis has two objectives:

The first objective is a pioneering step to furnish the theoretical basis behind the ACI 318 semi-empirical punching formula. Principles of the Boundary Element Method (BEM) and the theory of elasticity are adopted to establish the rationale behind the punching shear stress equation proposed in ACI 318. The present research further substantiates this effort through developing a BEM-based software for checking punching limit state for columns of any irregular cross-section shape – a task that ACI equation falls short of satisfying. The software is first verified via regular shape columns then adopted to demonstrate its capability to consider columns with irregular cross sections. Finally, some real-world examples are conducted to demonstrate the accuracy, efficiency, practicality and reliability of the proposed computational “theoretically-based” technique and developed software in checking punching of reinforced concrete columns with arbitrary (either regular or irregular) cross-sections.

The second objective is proposed to the ACI equation for checking punching a new term in order to include the effects of warping. A novel method for estimating the effects of warping of slab internal columns on punching stresses is introduced. In doing so, new punching parameters  $\gamma_{vw}$  and  $J_w$  are presented and evaluated/calibrated to include the effects of warping on punching and moment transfer analysis. The method is based on elastic analysis of the entire slab using principles of the theory of elasticity via the boundary element method (BEM) and the shear deformable plates. Applying the proposed method to some real-world examples, it has been shown that punching checks shall be carried out with warping effects considered in order to avoid un-conservative results from a design perspective, i.e., to avoid underestimation of actual maximum punching shear stress demand.

# **Chapter 1**

## **Introduction**

### **1.1 General**

Flat plates are one of the most commonly used structural systems nowadays. Flat plate/slab is a two dimensional solid reinforced concrete of uniform thickness in most cases that transfers the load directly to the columns without the need of projected beams. Architects prefer this system because the flexibility in the arrangement of columns and partitions with no obstruction of light. In flat slabs drops and column heads may appear at the vicinity of columns due to structural and architecture requirements.

Check of safety against punching two-way shear is an essential part of reinforced concrete flat slab design procedures. The significance of punching check increases in the analysis and design of high rise building slabs due to existence of additional values for unbalanced flexure moment and torsional moment resulted from lateral loading effect.

Although the fact that most of the codes provide conditions and precautions for flat slab's safety, there are still some points that are not precisely described. One of these points is the punching failure analysis in vicinity of slab-columns connection. In which this type of failure is a brittle failure. Also, the slab column connection subjected to high lateral loads fails due to lack in ductility. Therefore, many researchers focused on investigating this problem.

Some researchers used to study of the finite element method for the punching behavior. In such case, the studied problem is modeled using a finite element refined mesh. The results are obtained from the equilibrium of internal forces at mesh nodes. However, the use of the finite element method has some disadvantages such as; the results obtained depend on the mesh used in modeling of the problem. Another problem is that the appearance of stress concentration around the points where the column exists due to modeling of column as centerline frames. Also, the shape of irregular column is difficult to be modeled with the exact shape using the finite element method.

In the experimental method the boundary conditions of the slab are not given a high attention in studying the punching phenomenon. It is assumed that the slab boundaries are far away from the punching surface, therefore their influence can be neglected. Another factor is the high cost of testing a slab with realistic boundary conditions.

In general, both the experimental and the FEM are not practical methods for such analysis.

The use of the boundary element method overcomes several drawbacks of the FEM. Such as, the ease in defining and vary boundary elements, its accuracy and the real geometric representation of the column.

To overcome most of the previously mentioned problem in this thesis, a boundary element method for analysis of punching column-slab connections for any geometry of supports is used.

### **1.2 Previous Work**

The study of plate bending problems was thoroughly investigated in many research works. Due to the fact that plates have more complex geometry and/or loading in practical cases. The need to solve such cases derived researchers to seek the solution such as numerical methods.