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شبكة المعلومـــات الجامعية التوثيق الالكتروني والميكروفيا.



# جامعة عين شمس

التوثيق الالكتروني والميكروفيلم



نقسم بللله العظيم أن المادة التي تم توثيقها وتسجيلها على هذه الأفلام قد اعدت دون آية تغيرات



## يجب أن

تحفظ هذه الأفلام بعيداً عن الغبار

40-20 في درجة حرارة من 15-20 منوية ورطوبة نسبية من

To be kept away from dust in dry cool place of 15 – 25c and relative humidity 20-40 %









BINA



STATISTICAL ANALYSIS FOR LIFE EXPECTANCY
OF MANUFACTURED ITEMS AND ITS APPLICATION
FOR INVENTORY PLANNING OF SPARE PARTS

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## INTRODUCTION

The problem treated here is to find the optimal policy for keeping spares for machines which if they breakdown hold up production. This policy is based on knowledge of life expectancy of these spares.

The conditional failure rate of a life or failure distribution plays important role in the analysis. It is defined as

$$h(t) = f(t)$$
 (0-1)

where

f (t) is the failure density function

and

F (t) is the failure distribution = 
$$\int_{0}^{y} f(t) dt$$
.

An essential step in any inventory problem is to determine the distribution of the number of failures (replacements) in a fixed length of time t.

Assume a system of n equipments where only one equipment can fail at a time. The states of the system represent the number of equipment that fail. Thus the state o represents the state where all n equipments are operating, while the state n represents the state where non are operating. If we allow the system to start in state o, we are interested in how transitions are made to the successively higher states. We will make the following assumptions.

- The probability of a transition in interval t, t+dt is h(t) dt.
- 2 The probability of more than one failure in the interval t, t+dt is of order o(dt).
- 3 The transition probabilities are independent of the state of the system.

Therefore we can write the probabilities of being in each state  $0, 1, 2, \ldots, n$  at time t, t+dt as

$$P_{o}(t+dt) = P_{o}(t) [1 - h(t) dt] + o(dt)$$

$$P_{i}(t+dt) = P_{i+1}(t) h(t) dt + P_{i}(t) [1 - h(t) dt] + o(dt)$$

$$o < i < n$$

$$P_{n}(t+dt) = P_{n-1}(t) h(t) dt + P_{n}(t)$$

The system differential equations becomes

$$P_{0}^{*}(t) = -h(t) P_{0}(t)$$

$$P_{1}^{*}(t) = h(t) P_{1-1}(t) - h(t) P_{1}(t)$$

$$P_{n}^{*}(t) = h(t) P_{n-1}(t)$$
o < i < n

with the initial condition

$$P_0(0) = 1$$
  $P_1(0) = P_2(0) = \cdots = P_n(0) = 0$ 

The solution of these differential equations will provide us with the probability distribution for n failures up to time t. Using l'aplace transform and its inverse we find

where 
$$P_{0}(t) = e$$

$$H(t) = \int_{0}^{t} h(x) dx. \quad (o-2)$$
Then 
$$P_{1}(t) = H(t) e$$

and recursively we find

$$P_{n}(t) = \frac{\left[H(t)\right]^{n}}{n!} \quad e \quad (o-3)$$

which is a Poisson process.

If 
$$h(t) = \lambda$$
 then  $H(t) = \lambda t$   
and  $P_n(t) = \frac{\lambda t}{n!} = \frac{\lambda t}{n!}$ 

The investigation of the problem has been divided into Four major chapters.

Chapter 1 under the title "STATISTICAL ANALYSIS OF ITEM FAILURES" is primarily concerned with the types of failure and most attention has been focused on the concept of reliability as a criteria for the life expectancy of the spare parts. Based on component failure distribution the system reliability and expected time to system

failure are derived. The techniques for evaluating the probabilities properties of systems having identical operational and spare units with different facilities for repair are discussed. The important properties studied are

- 1) Reliability and transient behavior of systems.
- 2) The expected values of times to failures and the measure of variation.

Chapter 2 discusses the determination of optimum stock level. An explicit formula is given for calculating the minimum number of spare parts needed to achieve system reliability of at least  $\propto$ . Further analysis was done using a simulation model. The simulation model was directed toward improving control.

Chapter 3 is concerned with the mathematical solution for the computation of the spare parts which maximizes assurance of continued operation subject to a fixed budget for spares The functional equation technique of dynamic programming may be used to determine types of spares and quantities in order to construct the most reliable system subject to the given budget constraint. A numerical example is given to illustrate the computational procedure. This method appears best suited for computer evaluation.

Chapter 4 gives the practical application of the inventory models to planning the best level of stock holding of spares in a realistic inventory situation, where shortage cost is of much important than other types of inventory costs. We therefore take the reliability of the system as the more essential criterion for inventory policy of spares.

#### CHAPTER 1

## STATISTICAL ANALYSIS OF ITEM FAILURES

Three general types of failure can be distinguished which may occur without any fault on the part of the operator.

### 1 . 1 - Wearout failure:

A wearout failure results from gradual degradation of some property of the structural system essential to proper operation. The degradation may occur from causes such as fatigue. The probability of wearout failures increases with age extremely slowly at first but as the age approaches the mean wearout life the increase becomes very rapid. Normal failure distribution usually approximates wearout phenomena quite well. mal distribution has the difficulty that the area under the curve becomes 100 percent only if the curve is extended to infinity in both directions. This is not possible for time dependent events because a new equipment (component) enters service at T = 0 and not at  $T = -\infty$ The normal distribution can therefore be regarded as an approximation. However this approximation is very good in most cases especially when the standard deviation is small compared with the mean life.

Igor Bazovsky( $^*$ 6) has shown that when the mean life  $< 3 \circ (3 \text{ standard deviation})$  one would have to consider a transformation of the normal to the logarithmic normal distribution of the wearout failures. The latter has the advantage of having the value f(T) = 0 at T = 0

### 1.2 - Chance failure:

A chance failure occurs when the stress level in any critical part of the system exceeds the specified stress of the structural material. This type of failures has a fixed probability of occurring at any time. It generally result from severe unpredictable and usually unavoidable stresses that arise from environmental factors such as vibarations, temperature, shock and pressure.

Exponential distribution is the most appropriate model which describes the chance failure quite well since the failure rate in this type of failure is constant. In the exponential distribution the probability of failure is independent of the component age and therefore the same for a given time T.

To prove this we compute the probability of failure of an exponential component, in a t-hour, when the component has an age T which is 1 - e

But this is also equal to the cumulative probability of failure from o to t which proves that the exponential probability of failure in an arbitrary time interval t is independent of age T of the component.