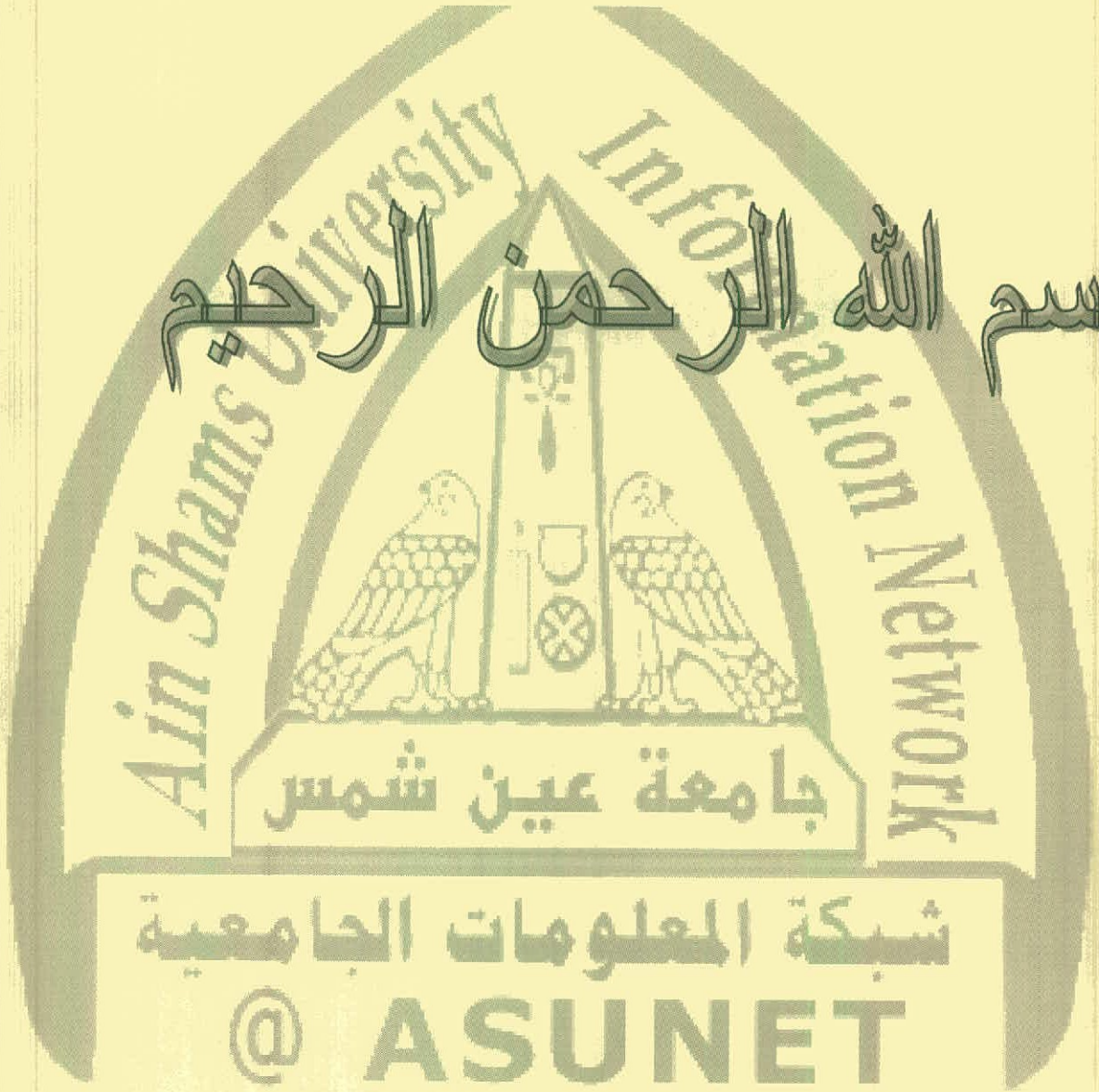
شبكة المعلومات الجامعية



بسم الله الرحمن الرحيم

شبكة المعلومات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها
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بعض الوثائـــــــق الاصليـــــة تالفـــة

بالرسالة صفحات لم ترد بالاصل

Mathematics Department
University College for women ,
Ain Shams University .

ON THE SOLUTIONS OF THE LINEAR DIFFERENTIAL EQUATIONS OF OPERATORS

THESIS

Submitted in partial fulfillment of
requirements for the degree
of
Master of Science
(pure mathematics)

By

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B. Sc. Mathematics 1992 .

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1995

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Afaf Rady.

CONTENTS

LIST OF FIGURES

INTRODUCTION

In the field of Mikusinski's operators†, we are led to differential equations which can not be solved in closed form . (By a solution in closed form we mean a solution expressed in terms of elementary functions, that is, polynomials, rational functions, exponential, logarithmic and trigonometric functions, etc., ...).

The theorems involving the equation

$$a_n(\lambda)\, x^{(n)}(\lambda) + a_{n-1}(\lambda)x^{(n-1)}(\lambda) + \ldots + a_0(\lambda)x(\lambda) = 0, 0 \le \lambda < \Lambda < \infty \quad \ldots\ldots \text{(I)}$$

where :

$a_i(\lambda)$, $i = 0,1,\ldots,n$ are independent of λ, are given by Drobbots and Mikusinski [6] .

B. Stankovic [18], [19] considered equation (I) Where $a_i(\lambda) \in C_s(\lambda)$ and proved the existence and the uniqueness of the solution in $C_s^{-\rho}(\lambda)$.

J. Mikusinski [8], [9] Solved equation (I) when $a_i(\lambda)$ are polynomials of s with numerical coefficients. Also, when $a_i(\lambda)$ are constant operators, he found that the corresponding characteristic equation not always has n-solutions. In fact he gave only the following example:

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† For the definition see chapter I.

The equation $\omega^2 = f$, where f = {t sin lnt}has no solution. Marija Skendzic [7] and I. E. Shrkawi [14] gave analytic solutions of (I) where $a_i(\lambda)$ belongs to certain class .

Also, the theory of operational differential equation* has not yet been satisfactory developed.

B. Stankovic [22] has proved the existence and the uniqueness of the solution of the initial value problem for the equation.

$$x'(\lambda)+a(\lambda)x(\lambda)=b(\lambda),0\le\lambda\le\Lambda \quad .....(II)$$

where: $a(\lambda)$, b(λ) belong to certain classes of functions [25] .

J. Mikusinski [8] investigated the exponential function exp. ($-\lambda\ s^{\alpha}$) where λ is real and $s^{\alpha}=\left\{\frac{t^{\alpha-1}}{\sqrt{\alpha}}\right\}$ for $\alpha<0$,$s^{-\alpha}=\frac{1}{s^{\alpha}}$ for $\alpha>0$ and $s^0=1$, which arises from solving the differential equation .

$$x'(\lambda)+\lambda\ s^{\alpha}x(\lambda)=0$$

* For the definition see chapter II

Some authors discussed the representation of the exponential function like . J. Mikusinski [10] , S. Osypow [11] and B. Stankovic [23] .

J. Mikusinski [8] proved that it is impossible to give a general definition of the exponential function in the field of Mikusinski's operators by means of power series. Since the series of the function exp. (- λs) with $\lambda > 0$, for instance , is not convergent and not locally integrable function.

J. Mikusinski [8] analyzed the exponential function exp. $(-\lambda \sqrt{s^2 + a^2})$. Also he found the analytic expression of exp. ($-\lambda \sqrt{s}$) as follows:

$$\text{exp. } (-\lambda\sqrt{s}) = \left\{ \frac{\lambda}{2\sqrt{\pi t^3}} \text{ exp. } (\frac{\lambda^2}{4t}) \right\}, \ 0 < \lambda < \infty$$

D. Nikolic [5] analyzed the exponential function

$$\text{exp. } \left(-\lambda \sqrt{s^m - \frac{\alpha}{s^n}} \right)$$

Our thesis is divided into three chapters. In the first chapter we shall be concerned with the concept and properties of convolution of continuous functions , and the field of