



شبكة المعلومات الجامعية

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Ain Shams University Information Network  
جامعة عين شمس

شبكة المعلومات الجامعية

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# شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم



شبكة المعلومات الجامعية

# جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

## قسم

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# بالرسالة صفحات نم ترد بالاصل

Mathematics Department  
University College for women ,  
Ain Shams University .

ON THE SOLUTIONS OF THE LINEAR DIFFERENTIAL  
EQUATIONS OF OPERATORS

B4995

THESIS

Submitted in partial fulfillment of  
requirements for the degree

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(pure mathematics)

By

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- |                          |               |
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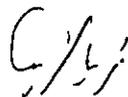
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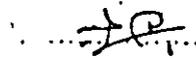
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Arabic Summary

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## INTRODUCTION

In the field of Mikusinski's operators<sup>†</sup>, we are led to differential equations which can not be solved in closed form . ( By a solution in closed form we mean a solution expressed in terms of elementary functions, that is, polynomials, rational functions, exponential, logarithmic and trigonometric functions, etc., .....).

The theorems involving the equation

$$a_n(\lambda) x^{(n)}(\lambda) + a_{n-1}(\lambda)x^{(n-1)}(\lambda) + \dots + a_0(\lambda)x(\lambda) = 0, 0 \leq \lambda < \Lambda < \infty \dots \text{ (I)}$$

where :

$a_i(\lambda)$ ,  $i = 0, 1, \dots, n$  are independent of  $\lambda$ , are given by Drobbots and Mikusinski [6] .

B. Stankovic [18], [19] considered equation (I) Where  $a_i(\lambda) \in C_s(\lambda)$  and proved the existence and the uniqueness of the solution in  $C_s^{-p}(\lambda)$ .

J. Mikusinski [8], [9] Solved equation ( I ) when  $a_i(\lambda)$  are polynomials of  $s$  with numerical coefficients. Also, when  $a_i(\lambda)$  are constant operators, he found that the corresponding characteristic equation not always has  $n$ -solutions. In fact he gave only the following example:

---

<sup>†</sup> For the definition see chapter I.

The equation  $\omega^2 = f$ , where  $f = \{t \sin \ln t\}$  has no solution. Marija Skendzic [7] and I. E. Shrkawi [14] gave analytic solutions of ( I ) where  $a_i(\lambda)$  belongs to certain class .

Also, the theory of operational differential equation\* has not yet been satisfactory developed.

B. Stankovic [22] has proved the existence and the uniqueness of the solution of the initial value problem for the equation.

$$x'(\lambda) + a(\lambda)x(\lambda) = b(\lambda), 0 \leq \lambda \leq \Lambda \dots\dots(II)$$

where:  $a(\lambda)$  ,  $b(\lambda)$  belong to certain classes of functions [25] .

J. Mikusinski [8] investigated the exponential function  $\exp. (-\lambda s^\alpha)$  where  $\lambda$  is real and  $s^\alpha = \left\{ \frac{t^{\alpha-1}}{\alpha} \right\}$  for  $\alpha < 0$  ,  $s^{-\alpha} = \frac{1}{s^\alpha}$  for  $\alpha > 0$  and  $s^0 = 1$  , which arises from solving the differential equation .

$$x'(\lambda) + \lambda s^\alpha x(\lambda) = 0$$

---

\* For the definition see chapter II

Some authors discussed the representation of the exponential function like .

J. Mikusinski [10] , S. Osypow [11] and B. Stankovic [23] .

J. Mikusinski [8] proved that it is impossible to give a general definition of the exponential function in the field of Mikusinski's operators by means of power series. Since the series of the function  $\exp. (-\lambda s)$  with  $\lambda > 0$  , for instance , is not convergent and not locally integrable function.

J. Mikusinski [8] analyzed the exponential function  $\exp. (-\lambda \sqrt{s^2 + a^2})$ . Also he found the analytic expression of  $\exp. (-\lambda \sqrt{s})$  as follows:

$$\exp. (-\lambda \sqrt{s}) = \left\{ \frac{\lambda}{2 \sqrt{\pi} t^3} \exp. \left( \frac{\lambda^2}{4t} \right) \right\}, 0 < \lambda < \infty$$

D. Nikolic [5] analyzed the exponential function

$$\exp. \left( -\lambda \sqrt{s^m - \frac{\alpha}{s^n}} \right)$$

Our thesis is divided into three chapters. In the first chapter we shall be concerned with the concept and properties of convolution of continuous functions , and the field of