



# MODELLING TRANSPORT IN FRACTURED MEDIA USING FRACTURE CONTINUUM APPROACH

# By Mohamed Ismaiel Mohamed Ahmed

A Thesis Submitted to the Faculty of Engineering at Cairo University In Partial Fulfillment of the Requirements for the Degree of MASTER OF SCIENCE

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#### IRRIGATION AND HYDRAULICS ENGINEERING

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Under the Supervision of

Prof. Ahmed Emam Ahmed Hassan	Dr. Mohamed Attia Mohamed
Professor of Hydrogeology	Assistant Professor
Irrigation and Hydraulics Department	Irrigation and Hydraulics Department
Faculty of Engineering, Cairo	Faculty of Engineering, Cairo
University	University

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Approved by the Examining Committee:

Prof. Dr. **Ahmed Emam Ahmed Hassan**, Thesis Main Advisor Professor of Hydrogeology Faculty of Engineering – Cairo University

Prof. Dr. **Abdulwahab Mohamed Amer**, Internal Examiner Professor of Hydraulics
Faculty of Engineering – Cairo University

Prof. Dr. **Ahmed Ali Hassan**, External Examiner Professor of Hydrogeology Faculty of Engineering – Ain Shams University

FACULTY OF ENGINEERING, CAIRO UNIVERSITY GIZA, EGYPT 2016 **Engineer's Name:** Mohamed Ismaiel Mohamed Ahmed

**Date of Birth:** 20/04/1991 **Nationality:** Egyptian

**E-mail:** mohamed.ismaiel@hotmail.com

**Phone:** 01112253539

**Address:** 34 Gamal El Din Abdallah St.- Haram-

Giza

**Registration Date:** 01/10/2012 **Awarding Date:** .../.../2016

**Degree:** Master of Science

**Department:** Irrigation and Hydraulics Engineering

**Supervisors:** 

Prof. Ahmed Emam Ahmed Hassan

Dr. Mohamed Attia Mohamed Abdelmegeed

**Examiners:** 

Prof. Ahmed Ali Hassan (External examiner)

Professor at Faculty of Engineering-Ain Shams University Prof. Abdulwahab Mohamed Amer (Internal examiner) Prof. Ahmed Emam Ahmed Hassan (Thesis main advisor)

#### **Title of Thesis:**

## Modelling Transport in Fractured Media Using Fracture Continuum Approach

#### **Key Words:**

Fracture Network; Fracture Continuum; Grid; Mapping; Conservative Transport.

#### **Summary:**

Fracture continuum arises recently as a fast and efficient technique to model fluid flow and solute transport in fractured geologic media. This study aims at developing a 2-dimensional model to solve the contaminant transport on the discrete fracture network as a reference solution. Additionally, developing a technique to map the fracture network onto a finite difference grid to solve conservative solute transport with preserving the transport characteristics for both fine and coarse grid resolutions when compared to the reference solution. The study is performed in a stochastic framework using Monte Carlo analysis. The results provide a conductivity correction factor to preserve the transport characteristics on the mapped domain for a high densely fracture network.



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#### **Abstract**

Fractured geological media are usually modelled using one of two approaches. The first is called Discrete Fracture Network (DFN) and the second is the Stochastic Continuum (SC) approach. DFN models are computationally demanding and require large input parameters, whereas SC approaches are computationally efficient and require fewer parameters for solving coupled flow and transport problems. Unfortunately, SC techniques do not preserve network details and properties.

Fracture continuum approaches (FC) combine the advantages of both the DFN and SC approaches. Fracture network details are preserved and a finite difference grid is then utilized for solving fluid flow by assigning a conductivity contrast between the grid cells representing the rock matrix and those representing fractures. Recent studies of FC focused on preserving flow properties on the mapped grid with less attention to transport properties. The main objective of this study is to develop an approach based on particle motion for solving transport on DFN, and to develop a mapping technique to preserve transport characteristics between DFN and mapped continuum. A two-dimensional particle tracking model is developed to simulate conservative contaminant transport through DFN. The model results are verified against an analytical solution for a single fracture and both analytical and numerical (SOLFRAC) solutions for a network of fractures.

The fracture network is randomly generated in a stochastic Monte Carlo framework, and the flow problem is solved using mass conservation at the fracture junctions. The transport problem is then solved via the developed particle tracking model on the DFN. The obtained flow and transport solutions are used as reference solutions. The fracture network is mapped onto a finite difference grid for four different grid cell sizes: (1m x 1m), (2m x 2m), (5m x 5m), and (10m x 10m), and the flow problem is solved via MODFLOW. The transport problem is solved on the grid using particle tracking method. Comparisons between the transport characteristics for both approaches (DFN and FC) are performed and the percentage error in each case is quantified. It is found that a new correction factor is needed to preserve conservative transport characteristics on the coarse grid. The correction factor could be developed using a small number of realizations, and subsequently applied for other realizations. The developed correction factor improves the ability of the FC technique to preserve transport on both fine grid and coarse grid for the case of conservative contaminant transport. Further efforts are needed to extend the work to more general, reactive contaminants.

### **Chapter One: Introduction**

### 1.1 Preamble

In the last two decades, stochastic modeling of subsurface coupled flow and contaminant transport has become a topic of wide interest and intensive research. This approach recognizes that hydrological variables are affected by the large degree of uncertainty due to the limited field data and consider them to follow statistical distribution based on the how each distribution fits these data. This randomness leads to defining models of flow and transport in a stochastic context, and predictions for both flow and transport are made in terms of probabilities rather than in the traditional deterministic framework.

This approach was adopted in many disciplines in water related field a long time ago. The closest field is of surface hydrology, which relies traditionally on time series analysis in predicting floods and other extreme events. However, subsurface modeling deals mainly with spatial variability, the uncertainty of which is of a more complex nature than many other fields. In addition, the physics of the phenomena is accounted for through the differential equations which is used to predict for flow and transport and various constitutive equations. These equations, regarded as stochastic, are intended to provide a general theoretical framework, rather than particular, empirical, statistical procedures. In this respect the subject is closer in outlook and methodology to the advanced statistical theories of continuum mechanics and of solid state physics.

However, the problem of modeling flow and transport in fractured geologic media continues to be a challenge. Fractures and fracture networks are conduits for migration of water and contaminants in groundwater systems. They also control movement of oil and gas in petroleum reservoirs. Fractures are also the principal pathways, through otherwise impermeable or low permeability rocks (Berkowitz 2002).

#### 1.2. **Problem Definition**

Fractured media are more challenging than the case of traditional porous media. Fractures have the main contribution towards the flow and transport phenomena. These two coupled processes are controlled by the main characteristics of the transmitting fractures. These characteristics include fracture orientation, length, aperture size, permeability, density and connectivity of fractures (Berkowitz 2002).

Two different techniques were developed over the past four decades for modeling flow and transport in the fractured media (Svensson (2001a), Svensson (2001b), Neuman (1987, 1988)). These techniques are Discrete Fracture Network (DFN) and Stochastic Continuum (SC) techniques. The DFN technique is based on modeling the fracture network as 1-D interconnected pipe network where flow and transport occur in this modeled pipe network. On the other hand, the SC approach is built on converting the whole domain to an equivalent porous medium with equivalent properties where the fracture network is discretized into grid cells and each cell is assigned a conductivity value based on the intersecting fractures.

Each technique has its own merits and drawbacks. The DFN is considered as the most accurate technique for solving and predicting flow and transport in the fractured media. However, it is computationally intensive as it requires a large number of input parameters to solve the domain depending on both domain size and density which cause this technique to be limited to small scale problems. SC is a computationally fast technique as it requires a fewer input parameters than the DFN needs. However, problem in defining the Representative Elementary Volume (REV) leads to unexpected results in both flow and transport. SC technique tends to over-homogenize the domain of study, and therefore, the flow and transport behavior differs from the expected real conditions.

Fracture Continuum (FC) technique arises as a new technique which combines the advantages of both DFN and SC and overcomes the disadvantages (Svensson 2001; Botros et al. 2008; Reeves et al. 2008). It is a fast and accurate technique, in which the domain of interest is mapped onto a finite difference grid with any desired cell size. Each cell is assigned an equivalent conductivity value based on the intersecting fractures with a correction factor to preserve the total flux along the whole area of study.

In recent studies, the focus was on mapping fracture network onto continuum such that the flow characteristics are preserved. Little attention was given to preserving transport characteristics in both fine grid (small cell size) and coarse grid (large cell size) compared to the original fracture network transport.

### 1.3. Research Objectives

The main goal of this study is to develop an approach to map fracture networks representing fractured geologic media into finite difference grid with different cell sizes in such a way that preserves conservative transport characteristics.

This can be achieved by addressing the following specific objectives:

- 1. Developing a technique for solving contaminant transport on DFN as a reference solution
- 2. Adopting a recently proposed flow-based FC technique to preserve transport on the mapped domain
- 3. Modifying grid transport technique to efficiently work with the mapped domain
- 4. Quantifying the error resulting from mapping and upscaling processes

### 1.4. Dissertation Scope and Methodology

A number of research tasks have been undertaken to achieve these operational objectives. The objectives are studied through the following general tasks:

- 1. Synthetic model generation with domain boundaries and internal fracture network generation based on fracture statistics
- 2. Trimming dead ends (eliminating zero flow segments from the domain of study)
- 3. Solving the flow and transport problems using the DFN technique as a reference solution.
- 4. Mapping fracture network onto a finite difference grid with the new proposed FC correction factor to preserve transport on the mapped domain.
- 5. Solving the mapped domain using the modified grid based transport model.
- 6. Quantifying the error resulting from the mapping and upscaling processes.
- 7. Applying a second correction factor to the mapping technique to preserve the transport properties for the coarse grid cell size only.

The study objectives are addressed in a stochastic framework using a Monte Carlo analysis. Various realizations are simulated with varying the controlling properties of the fracture network (fracture length, orientation, transmissivity, aperture size, density and connectivity).