Cairo University
Institute of statistical studies & research

An Ant Colony Optimization Approach for Solving Shortest Path Problem with Fuzzy Constraints

Master's thesis in operations research

Submitted by

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Summary

This thesis presents an Ant Colony Optimization Approach (ACO) to solve the shortest path problem, especially with fuzzy constraints, which is a new trend in dealing with the fuzziness of the shortest path problem and also in solving with ant colony optimization approach.

The proposed algorithm consists of five sequential steps. The first step is to determine the number of possible paths from the source to the target. The second step calculates the probability of each path of possible paths. The third step calculates the expected number of ants through each path of possible paths then calculates in the fourth step the new trail of each weight component for eachpath of possible paths, which leads to the final step to calculate the average trail of each path , therefore the shortest path is that one which have more trail through it.

The shortest path Problem is an NP-hard combinatorial optimization problem that has long challenged researchers. The objective of the shortest path problem is to find the path between two nodes with shortest length (weight).

This thesis consists of five chapters as follows:

Chapter One: we introduce an introduction, summary and review of literature.

Chapter Two: we handle the shortest path problem as definitions, formulation and the algorithms which solve it with some applications.

Chapter Three: an ant colony optimization approach is introduced as a methodology for solving the shortest path problem with some related definitions, applications, explanation of the general ant colony optimization (ACO) method and an illustrated example.

Chapter Four: we introduce an introduction on fuzzy sets, definitions, formulation algorithms which solve the fuzzy shortest path problem.

<u>Chapter Five:</u> we introduce the supposed algorithm in details with a flowchart represent it, a program was coded by JAVA programming for making an implementation study, finally we have got two empirical formulas from an experimental data to calculate the no of ants and the evaporation coefficient which give the best convergence to the optimal solution.

جامعة القاهرة
معهد الدراسات والبحوث الاحصائية

حل مشكلة المسار الأقصر ذات القيود المبهمة باستخدام منهجية مستعمرة النمل

للحصول على درجة الماجستير فى بحوث العمليات

مقدمة من

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ملخص الرسالة

هذة الرسالة تقدم منهجية مستعمرة النمل المثلى كمنهجية جديدة لحل مشكلة الطريق الأقصر ذات القيود المبهمة ،وهى إتجاه جديد لم يطرق من قبل فى التعامل مع القيود المبهمة فى المشكلة وكذلك فى إستخدام منهجية مستعمرة النمل فى الحل. تتكون الطريقة المقترحة من خمس خطوات وهى كالتالى:

١- تحديد الطرق الممكنة من المصدر (بداية التحرك) الى الهدف (الموقع المراد الذهاب اليه).
٢- إحتساب احتمال اتخاذ النمل لكل مسار من هذه المسارات الممكنة للسير فيها.
٣- إحتساب عدد النمل المتوقع سيره عبر كل مسار من هذه المسارات.
٤- حساب كمية الفيرومون التى يفرزها النمل عبر كل مسار من المسارات مراعيا دالة العضوية لقيم القيود المبهمة.
٥- حساب متوسط كمية الفيرومون عبر كل مسار من المسارات ويكون الطريق الأقصر هو الطريق الذى يحتوى على كمية فيرومون أكثر من بقية الطرق.

تتكون الرسالة من خمس فصول وهى كالتالى:

الفصل الأول: نقدم مقدمة الرسالة وملخص لها ثم بعض الأبحاث التى تناولت موضوع البحث.

الفصل الثانى: نقدم تعريف لمشكلة الطريق الأقصر مع بعض التعريفات المتعلقة بها وكذلك الشكل الرياضى للمشكلة وعرض لبعض الطرق المستخدمة فى حلها.

الفصل الثالث: نبين كيفية استخدام منهجية مستعمرة النمل المثلى كطريقة لحل مشكلة المسار الأقصر مع بعض التعريفات المتعلقة بالطريقة وكذلك تطبيقاتها وخطواتها.

الفصل الرابع: نقدم مراجعة على مفهوم المنطق المبهم وكيفية وجوده فى مشكلة الطريق الأقصر مع الشكل الرياضى له والطرق المستخدمة لحل مشكلة الطريق الأقصر ذات الشروط المبهمة مع تطبيقاتها.

الفصل الخامس: نقدم منهجية مستعمرة النمل المثلى كطريقة جديدة لحل مشكلة الطريق الأقصر ذات القيود المبهمة ، كما تم حل بعض المشكلات – والتى تم حلها بطرق أخرى من قبل بعض الباحثين – باستخدام الطريقة موضوع البحث وتم عمل دراسة كاملة لتأثير بعض المتغيرات فى عمل بعض التحسن فى الحل وتوفير الوقت اللازم للوصول للحل الأمثل وبتقارب سريع فتوصلنا لمعادلتين تم استنتاجهما من البيانات التجريبية والاحصاءآت الخاصة بالدراسة ساهمتا فى عملية الوصول للحل الأمثل وبتقارب سريع وفى وقت وجهد أقل وقد أثبتت الدراسة فعالية وكفاءة الطريقة المقترحة وتميزها فى النتائج عن الطرق المستخدمة فى حل المشكلة من قبل.

Chapter 1

Introduction and review of literature

1.1 Introduction

We introduce an ant colony optimization approach as a methodology for solving the shortest path problem with fuzzy constraints , this problem is to find the fuzzy shortest path (weight) from source to target , this problem is one of the optimization problems which have a high importance for the industrial world as well as for the scientific world. The practical optimization problems include for example train scheduling, time tabling, shape optimization, telecommunication network design, or problems from computational biology. The research community has simplified many of these problems in order to obtain scientific test cases such as the well-known traveling salesman problem (TSP). The TSP models the situation of a traveling salesman who is required to pass through a number of cities. The goal of the traveling salesman is to traverse these cities (visiting each city exactly once) so that the total traveling distance is minimal. Another example is the problem of protein folding, which is one of the most challenging problems in computational biology, molecular biology, biochemistry and physics. It consists of finding the functional shape or conformation of a protein in two or three dimensional space, for example, under simplified lattice models such as the hydrophobic polar model

The TSP and the protein folding problem under lattice models belong to an important class of optimization problems known as combinatorial optimization (CO) [6].

A CO problem P = (S, f) is an optimization problem in which are given a finite set of objects S (also called the search space) and an objective function f: S $\longrightarrow$ R^+ that assigns a positive cost value to each of the objects. The goal is to find an object of minimal cost value. The objects are typically integer numbers, subsets of a set of items, permutations of a set of items, or graph structures. CO problems can be modeled as discrete optimization problems in which the search space is defined over a set of decision variables with discrete domains. Therefore, we will henceforth use the terms CO problem and discrete optimization problem interchangeably.

Due to the practical importance of CO problems, many algorithms to tackle them have been developed. These algorithms can be classified as either complete or approximate algorithms. Complete algorithms are guaranteed to find for every finite size instance of a CO problem an optimal solution in bounded time. Yet, for CO problems that are non polynomial (NP) hard problems, no polynomial time algorithm exists, therefore, the complete methods might need exponential computation time in the worst-case. This often leads to computation times too high for practical purposes. Thus, the development of approximate methods—in which we sacrifice the guarantee of finding optimal solutions for the sake of getting good solutions in a significantly reduced amount of time—has received more and more attention in the last 30 years [6].

1.2 Summary

This thesis presents an Ant Colony Optimization Approach (ACO) to solve the shortest path problem, especially with fuzzy constraints, which is a new trend in dealing with the fuzziness of the shortest path problem and also in solving with ant colony optimization approach. The proposed algorithm consists of five sequential steps. The first step is to determine the number of possible paths from the source to the target. The second step calculates the probability of each path of possible paths. The third step calculates the expected number of ants through each path of possible paths then calculates in the fourth step the new trail of each weight component for ach path of possible paths, which leads to the final step to calculate the average trail of each path , therefore the shortest path is that one which have more trail through it.

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1.3 Review of literature

The shortest path problem with certain arc lengths has been studied intensively. And many efficient algorithms have been developed by Ronald Gould [31], Liewellyn [26], and these algorithms are referred to as the standard shortest path algorithms.

However, due to failure, maintenance or other reasons, different kinds of uncertainties are frequently encountered in practice, and must be taken into account. For example, the lengths of the arcs are assumed to represent transportation time or cost rather than the geographical distances, as time or cost fluctuate with traffic or weather conditions, payload and so on, it is not practical to consider each arc as a deterministic value. In these cases, probability theory has been used to attack randomness, Ronald Gould [31] studied the probability distribution of the shortest path length in which arc lengths are random variables. Ronald Gould [31] and Liewellyn [26] considered the different types of cost functions to study the variations of the shortest path problem in stochastic networks. Lady [11] studied the expected shortest paths in dynamic and stochastic networks.

Several algorithms have been proposed over the years to handle efficiently restricted families of weighted directed graphs, such as, for instance, directed graphs with nonnegative weight functions or acyclic graphs [35].

A solution to the shortest path problem is sometimes called a pathing algorithm. The most important algorithms for solving this problem are [37].

Marco Dorigo [24] introduced the first ant colony optimization algorithms. the development of these algorithms was inspired by the observation of ant colonies.

After the initial proof-of-concept application to the traveling salesman problem

(TSP), ant colony optimization (ACO) was applied to many other problems, such as the Assignment problems, scheduling problems and vehicle routing problems. Among other applications, ACO algorithms are currently state-of-the-art for solving the sequential ordering problem, the resource constraint project scheduling problem, and the open shop scheduling problem [38] , Kourosh and Morteza [21] have recently published an excellent book in Aco in which the reader can find a comprehensive review of different versions of ACO algorithms. One of ACO versions, called Ant Colony System is used.

M.Blue examined the problem of Boolean flows on a fuzzy network [29], also, Tzung [40] presented a method based on Floyd's algorithm and Ford's algorithm to treat the fuzzy shortest path problem. It can obtain the shortest path length whereas the corresponding shortest path in the network perhaps does not exist. Later, Tzung [40] proposed an improved algorithm that was based on dynamical programming recursion. It canget not only the shortest path length but also the corresponding shortest path in the network; nevertheless, the assumption that the possible arc lengths are 1 through a fixed integer seems to be impractical. Moreover there is who solve the fuzzy shortest path problem such as Yinzhen [43] who solve the problem by neural networks, also Yanbin [42] introduced an improved ant colony optimization algorithm for solving another problem (mobile agent routing problem), so it is a new trend to solve the shortest path problem with fuzzy constraints using the ant colony optimization approach.

Chapter 2

The shortest path problem

2.1 Introduction

Shortest path problems are among the fundamental problems studied in computational geometry and other areas including graph algorithms, geographical information systems (GIS), network optimization and robotics.

There are two types of the shortest path problem, the first type is called The geodesic shortest path problem which given two points s and t on the surface of a polyhedron, find the shortest path on the surface from s to t.

The second type is called the Euclidean shortest path problem and is looking for the shortest path among the obstacles in 3D space. Whereas finding the Euclidean shortest path is non polynomial hard problem, the geodesic shortest path may be found in polynomial time.

The shortest path problem can be next categorized by the distance measure used (Euclidean, weighted), purpose *(single source shortest path proble*m: the shortest path between two points or *all pairs shortest path problem:* the shortest paths between one point and all triangle vertices) and computation *(exactly, approximate*) [8].

A network or graph (see figure 2.1) consists of arcs (links) and nodes (vertices) so the shortest path problem is concerned with finding the shortest path in that network from one node (or set of nodes) to another node (or set of nodes) [19] .

In graph theory, given a source node s in a weighted directed graph G, with n nodes and m arcs, the single-source shortest path problem (SSSP) from s is the problem of finding the minimum weight paths from s to all other nodes of G [35].

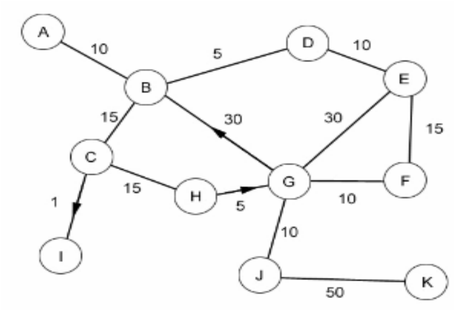

Fig 2.1 An example of graph

Then the shortest path problem, finding the path with minimum distance, time or cost from a source to a destination, is one of the most fundamental problems in network theory. It arises in a wide variety of scientific and engineering problem settings, both as stand-alone models and as sub problems in more complex problem settings.

However, due to failure, maintenance or other reasons, different kinds of uncertainties are frequently encountered in practice, and must be taken into account. For example, the lengths of the arcs are assumed to represent transportation time or cost rather than the geographical distances, as time or cost fluctuate with traffic or weather conditions, payload and so on, it is not practical to consider each arc as a deterministic value.

The objective of the resource constrained shortest path problem (RCSP) is to find the shortest path in a network such that certain constraints are satisfied.

RCSP is NP-hard, Good heuristics are thus required so that on a practical problem size an answer may be found in a reasonable amount of time. This makes RCSP an interesting problem in general .

A solution to RCSP requires a graph algorithm. These are considered difficult to implement efficiently in a purely functional language .

The importance of computation of shortest paths arises rapidly, so it became one of the most fundamental problems in network analysis . These have been the subject of extensive research [31] .

The different types of the shortest path problem can be aggregated as:

1- One node-to-one node.

2- One node-to-some nodes.

3- One node-to-all other nodes.

4- All nodes-to-one node.

5- All nodes-to-all nodes.

If arcs in the network have non-negative values , a labeling algorithm can be used to find the shortest paths from a particular node to all other nodes in the network .

The criterion to be minimized is not limited to distance even though the term shortest path is used in describing the procedure , other criteria include time and cost (neither time nor cost are necessary linearly related to distances) [35].

The majority of published research on shortest paths algorithms has dealt with static networks that have fixed topology and fixed criteria (distances or costs) .

In solving static network shortest path problems, it usually aggregates a once-off all-to-all calculation for all nodes , from which subsequent paths then are derived, but it is not feasible for dynamic networks where the criteria (cost) is time-dependent or randomly varying , so one way of dealing with dynamic networks is splitting continuous time into discrete time intervals with fixed criteria.

Thus understanding shortest path algorithms in static networks becomes fundamental and necessary to work with dynamic networks [37].

Initially, elementary definitions will be introduced which can help us to understand the shortest path problem and its algorithms.

2.2 Graph Definitions

2.2.1 Vertex, vertices :

A **Vertex** of a Graph is a connection point and it may have no connections, one connection or many connections.

A **Graph** has a set of Vertices, usually shown as V = {v_1, v_2... v_n} or V = {A, B, C} or V = {1, 2 ... W}.

An **Edge** in a graph is a connection between vertices [31].

Given vertices v_1 and v_2 in a Graph, the edge between them may be written as (v_1, v_2) or sometimes [v_1,v_2].

An **Undirected Graph** has no implied direction of its edges.

A **directed Graph** has a direction of its edges.

A **Weighted Graph** has edges with an additional property, a weight, weights may be integers, real numbers, gallons per minute or any type of quantity.

A general graph may have a self loop which is an edge that goes from a vertex to itself, e.g. (v_3, v_3).

A **multi-graph** may have more than one edge between the same pair of vertices.

A **cutset** (one word) is a set of edges, which when removed, disconnect source vertices from sink vertices.

The **Edge Connectivity** of an Undirected Graph is the minimum number of edges that must be removed to disconnect the graph [31].

In the following figure 2.2, the graph's types will be introduced and approaches with scientists.

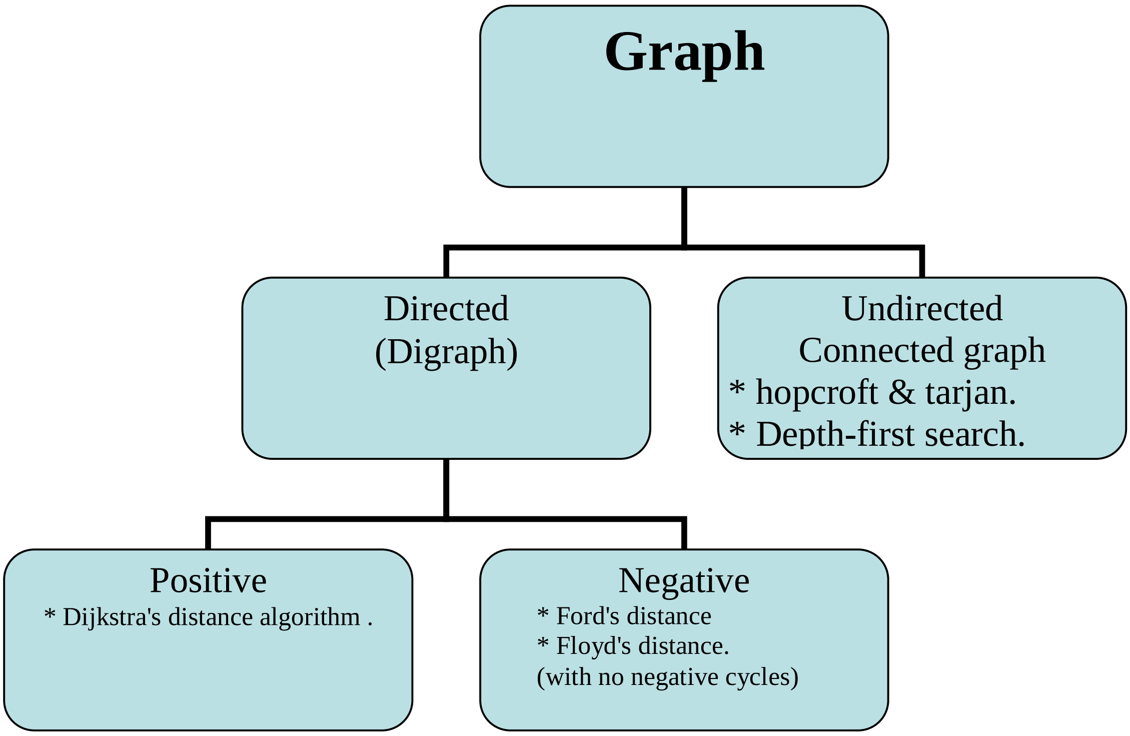

Fig 2.2 Graph's types and approaches with scientists.

Acyclic - no cycles, there is no Path that includes at least one edge that can return to the starting vertex.

Planar - The Graph can be drawn on a plane (paper, flat surface) with no Edge crossing another Edge. Note: "can be drawn" but it may be drawn with edges crossing.

Non planar - A Graph that is impossible to draw on a plane.

Connected - There are no unconnected vertices and no isolated groups of vertices. In an Undirected Graph there is a Path from every vertex to every other vertex.

Tree - a restricted form of a graph where one vertex is called the root and all vertices have a Path to the root and the graph is undirected and acyclic.

Bipartite - a Graph with a set of vertices that can be divided into exactly two non empty subsets such that no edge connects two vertices within a subset and every vertex in one subset has at least one Edge to a vertex in the other subset [26].