



شبكة المعلومات الجامعية

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شبكة المعلومات الجامعية

التوثيق الالكتروني والميكرو فيلم

جامعة عين شمس

التوثيق الالكتروني والميكرو فيلم

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PROJECTION CONSTANT OF SOME TOPOLOGICAL SPACES

THESIS

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By

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For
All of my family members
My parents

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