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PROJECTION CONSTANT OF SOME TOPOLOGICAL SPACES

THESIS

Submitted for the Award of the Degree of Doctor of Philosophy (Ph. D.) in Science (Pure Mathematics, Functional Analysis)

By

Mrs: Sahar Mohamed Ali Abo Bakr

Under the Supervision of

Prof. Dr. Entisarat M.H.El-Shobaky, Prof. Dr. Wataru Takahashi

Department of Mathematics,

Faculty of Science,

Ain Shams University,

Cairo, Egypt.

Department of Mathematical and Computing Sciences,

Tokyo Institute of Technology,

Tokyo, Japan

And

Dr. Ismail Kaoud Yousef
Department of Mathematics
Faculty of Science,
Ain Shams University,

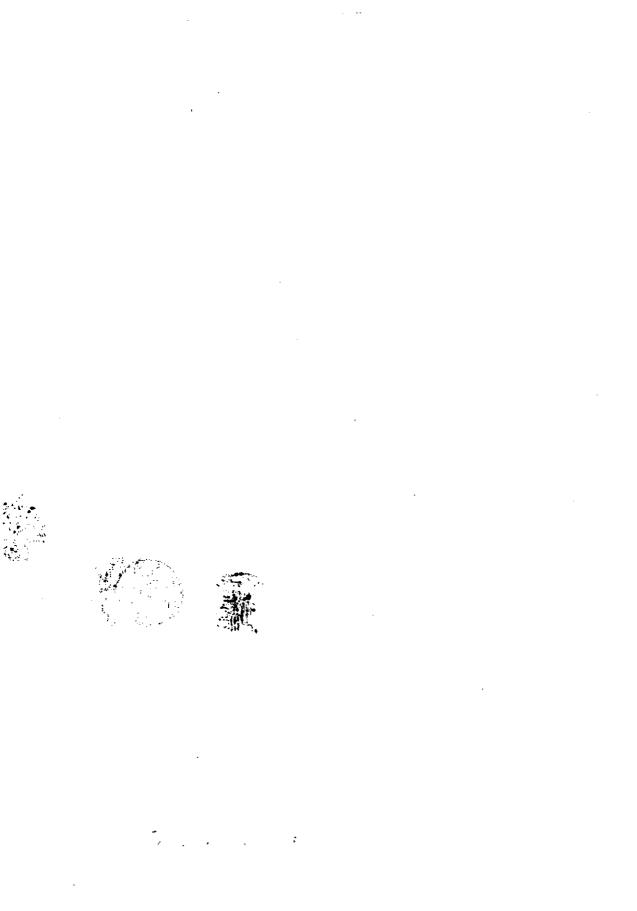
Cairo, Egypt.



Submitted to
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Faculty of Science
Ain Shams University
CAIRO, EGYPT.

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For All of my family members My parents

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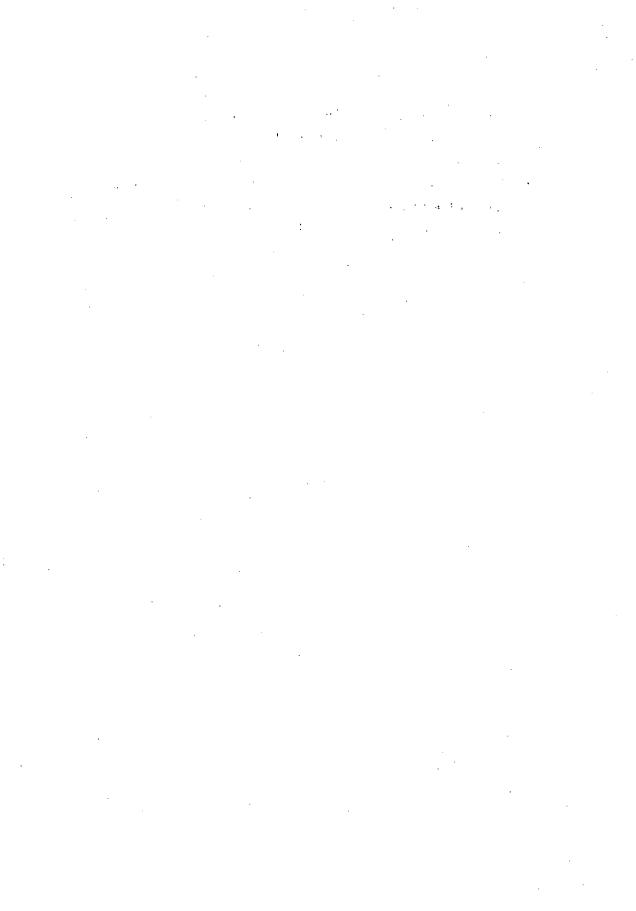
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Sahar Mohamed Ali
Mathematics Department,
Faculty of Science,
Ain Shams University.



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