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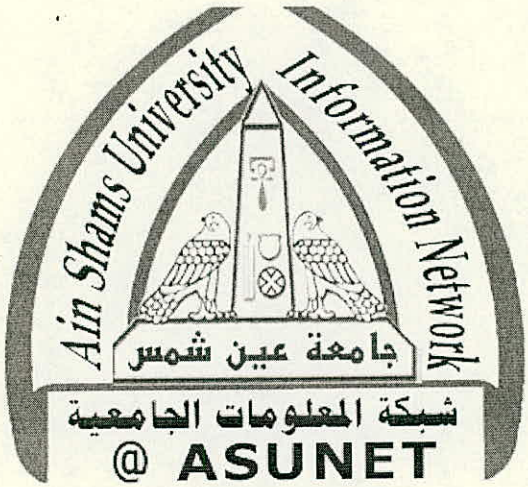

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RELATIVE INVOLUTIVE INVARIANTS OF RINGS

Thesis
Submitted for the Degree of
Doctor of Philosophy
in Science (Pure Mathematics)

By

SAFAA MOHAMED MAHMOUD MOUSTAFA

Department of Mathematics
Faculty of Education
Ain Shams University

SUPERVISORS

Prof. Dr. A. Verschoren
Department of Mathematics & Computer Science
RUCA University
Antwerp, Belgium

Prof. Dr. A. A. Dabbour
Department of Mathematics
Faculty of Science
Ain Shams University

Prof. Dr. E. M. El-Ghazzy
Department of Mathematics
Faculty of Education
Ain Shams University

Prof. Dr. A. G. El-Sakka
Department of Mathematics
Faculty of Science
Ain Shams University

To
Department of Mathematics
Faculty of Science
Ain Shams University
Cairo, Egypt
1998

To the memory of

my

MOTHER

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"FIRST AND FORMOST THANKS ARE TO **ALLAH**, *THE MOST BENFICENT AND MERCIFUL"*

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Contents.

SUMMARY

Summary.

Introducing the contents of this thesis is impossible without referring to the fundamental work of *A. Verschoren and F. Van Oystaeyen* [42], who introduced and studied the theory of relative invariants of commutative rings. One of the fundamental facts on which that theory has been built is that to infer knowledge about the structure of the ring from knowledge about certain invariants. Global invariants like the Picard group or the Brauer group seldom suffice to characterize the base ring, so it is usually necessary to consider more complicated invariants in order to get closer to our goal of obtaining useful structure results for the rings studied. In this thesis we develope an involutive version of the theory of relative invariants, both from an algebraic and a geometric point of view.

One of the classical invariants, as we just mentioned, is the Picard group. The category theoretical definition of the Picard group of an abelian category and its elementary properties are well known, [7]. In the module case, however, one may describe this Picard group more explcitly in terms of equivalence classes of invertible projective modules. After that the *Gabriel Popescu* theorem tells us that any Grothendieck category is equivalent to a quotient category of some module category R-mod with respect to some idempotent kernel functor σ, [17]. Thus one may hope to describe the Picard group of R-**mod** in terms of invertible modules, but involving the torsion at σ. This is done by *A. Verschoren* in [44]. In this thesis we aim to study the involutive version of the relative Picard group. We start by studying the notion of σ-discriminant module. This

concept is then used to study the so-called σ-discriminant group; which is shown to coincide with the relative involutive Picard group. In the relative case, the relative involutive Picard group yields the involutive class group, [35], when restricting attention to Krull domains.

On the other hand, why are we interested in algebras with involution at all? The answer is easy: because they appear in almost every branch of mathematics, and whenever appearing, they tend to play a prominent role.

Let us mention an example. It is well known that the Brauer group of a ring is a torsion group, so it is clear that one tries to get some grasp on the group $Br(R)$ by calculating each of its p-torsion subgroups $Br_p(R)$. In this context, algebras with involution come into the picture, due to *Saltman's* result [36], which states that every element in $Br_2(R)$ may be represented by an Azumaya algebra with involution, a result that was given a more polished treatment later by *Knus, Parimala and Srinivas* [24] and a geometric interpretion in [34]. In this thesis, we study the relative involutive Brauer group of any commutative ring with respect to an idempotent kernel functor σ. In the global case, this leads to the involutive Brauer group studied by *Parimala* and *Srinivas* and by *Reyes* and *Verschoren*. In the relative case the relative involutive Brauer group yields the involutive divisorial Brauer group, [35], when restricting to Krull domains.

Let us now give a brief description of the contents of this thesis.

The first chapter provides the necessary background on "abstract" lo-