



**VIBRATIONS ANALYSIS OF LAMINATED COMPOSITE AND EULER
BERNOULLI BEAMS USING DIFFERENTIAL TRANSFORMATION METHOD**

By

Eng. SOUMA MOHAMED ABDEL-GHANI MOHAMED

**A Thesis Submitted to the
Faculty of Engineering at Cairo University
In Partial Fulfillment of the
Requirements for the Degree of**

DOCTOR OF PHILOSOPHY

IN

ENGINEERING MECHANICS

**FACULTY OF ENGINEERING, CAIRO UNIVERSITY
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Title of Thesis : VIBRATIONS ANALYSIS OF LAMINATED COMPOSITE
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TRANSFORMATION METHOD

Key Words : Vibrations; Laminated beams; Composite beams;
Euler Bernoulli beams; Differential Transformation Method

Summary :

Differential Transformation Method is a technique that uses Taylor series for the solution of differential equations in the form of a polynomial. It is different from the high-order Taylor series method, which requires symbolic computation of the necessary derivatives of the data functions. The Differential Transformation Method leads to an iterative procedure for obtaining an analytic series solutions of functional equations. Beams with abrupt changes of cross-section are used widely in engineering. Rectangular and circular shapes of the cross section are chosen because it often occurs in practice. In this thesis, the vibration problem of Euler-Bernoulli beams has been solved using Differential Transformation Method. The beam has variable cross sections and various end conditions.

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NOMENCLATURE

<i>Symbol</i>	unit	Identification
E	GPa	 Young's modulus of the beam material
I	m^4	 Inertia of the beam
v	M	 Vertical displacement of the beam
A	m^2	 Area of the beam section
h	M	 Height of the beam
b	M	 Width of the beam
L	M	 Total length of the beam
ρ	Kg/m^3	 Density of the beam material
T	J	 Kinetic energy
V	J	 Strain energy
ω	Hz	 Beam natural frequency
Ω	-	 Non-dimensional natural frequency
$\bar{k}_1$	N/m^2	 Linear parameter of the foundation
$\bar{k}_3$	N/m^4	 Non-Linear parameter of the foundation
$\bar{\mu}$	$N S/m^2$	 Damping parameter of the foundation
δ	-	 Dirac delta function
$\bar{Q}$	KN	 Moving load magnitude
$\bar{v}$	m/s	 Moving load Velocity
t		 Time
T_n	S	 Non-dimensional time
M_x	N m	 Bending moment
M_{xy}	N m	 Torque
N_x	N	 Normal force
Q_{xz}	N	 Shear force
k_s	-	 Shear correction factor
θ	degree	 Angle betw. fiber directions and longitudinal axis of laminated beam
ν_{12}	-	 Possion's ratio
G_{12}	GPa	 Shear modulouls of elasticity
β	-	 Coefficient of variation of beam cross section
N	-	 Terms number
M	-	 Terms number
k	-	 Index
E_1	GPa	 Young's modulus of the laminated beam material in the fiber direction.
E_2	GPa	 Young's modulus of the laminated beam material in the perpendicular on the fiber direction.
Q_{ij}	-	... The transformed reduced stiffness constants of laminated beam
A_{ij}	-	 The extensional stiffness constants of laminated beam
B_{ij}	-	 The coupling stiffness constants of laminated beam
D_{ij}	-	 The bending stiffness constants of laminated beam