شبكة المعلومات الجامعية

بسم الله الرحمن الرحيم

شبكة المعلومات الجامعية

شبكة المعلومات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

قسم

نقسم باللّه العظيم أن المادة التي تم توثيقها وتسجيلها
علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات

يجب أن

تحفظ هذه الأقراص المدمجة بعيدا عن الغبار

بعض الوثائق الأصلية تالفة

شبكة المعلومات الجامعية

بالرسالة صفحات
لم ترد بالأصل

CONVEXITY STUDY IN RIEMANNIAN MANIFOLDS

THESIS

SUBMITTED TO DEPARTMENT OF MATHEMATICS

FACULTY OF SCIENCE, TANTA UNIVERSITY

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

IN

(Pure Mathematics)

BY

Ibrahim Ahmed El-Bastawesy Sakr

Assistant Lecturer, Mathematics Department

Faculty of Engineering-Shoubra, Zagazig University

Under the Supervision of

Dr. M. Beltagy

Professor of Pure Mathematics

Faculty of Science,

Tanta University

Dr. M. I. M. Hessein

Professor of Pure Mathematics

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Head of Mathematics Department

(Prof. Dr. M. O. Shaker)

To

My Mother

The Memory of my Father and my Brother

To My Wife

To My Family

CURRICULUM VITAE

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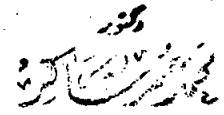

(Prof. Dr. M. O. Shaker)

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CONTENTS

Preface

PREFACE

This thesis includes a study of the necessary and sufficient conditions for convexity of subsets in general Riemannian manifolds with applications in special types of manifolds such as Euclidean, hyperbolic and elliptic spaces.

The whole thesis consists of four chapters, an introduction (Chap. I) together with three chapters (Chap. II, Chap. III, Chap. IV) which contain the main results we have established.

In Chapter I, we have quoted the necessary background material for the following three chapters. Accordingly, we wrote few sections on manifolds, submanifolds, Riemannian manifolds, connexions, convexity and forms, ..., etc, which are important for our study.

In (1990), D. mejia and D. Minda established the concept of K-convex region Ω with boundary $\partial\Omega$ in Euclidean 2-space E^2 [21]. The main aim of Chapter II is to define and study the concept of k-convexity of regions in the Euclidean 3-space E^3. In this chapter we established some results relating the k-convexity property to other geometric characteristics such as the sectional curvature as well as the focal points of the boundary of the considered region. The main results of this chapter have been accepted for publication in the

"Communications of Faculty of Science, University of Ankara" Vol. 44 (1995) under the same title.

Chapter III has been devoted to deal with another type of convexity called uniform convexity. This chapter consists of two sections A and B.

In (1979), K. P. R. Sastry and S. V. R. Naidu [24] proved a remarkable result which gives a sufficient condition for convexity of a subset of a topological vector space. Also they defined and studied uniformly convex subsets of metric linear spaces.

In Section A (Chapter III) we established some necessary and sufficient conditions for convexity as well as strict convexity of subsets in different types of Riemannian manifolds. The main results of this section have been published in Delta J. of Science 18 (3), (1994), 11-18.

In Section B (Chapter III), the uniform convexity as well as strict uniform convexity have been defined and studied for subsets of Riemannian manifolds. We have established the necessary and sufficient conditions for a Riemannian manifold without conjugate points to be free from focal points in terms of the uniform convexity concept. The main results of this section have been accepted for publication in the Communications of the Faculty of Sciences, Ankara University, Vol. (45) (1996).