Al-Azhar University
Faculty of Commerce
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BAYESIAN AND NON BAYESIAN APPROACH UNDER ACCELERATED LIFE TESTING FOR GENERALIZED INVERSE GAUSSIAN DISTRIBUTION

A Dissertation Submitted to Al-Azhar University, Faculty of Commerce, Girls' Branch in Partial Fulfillment for the Requirements of the Degree of Ph.D

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1435 A.H.-2014 A.D

ACKNOWLEDGMENT

My gratitude, first of all, is due to ALLAH for inspiring me to complete this thesis.

It is impossible to thank individually the many people who have contributed directly and indirectly to make this thesis possible. I am grateful to them all, but I would like to mention some in particular.

I wish to express my deepest gratitude to prof. **Shaban, A. Shaban, Prof. of Statistics,** Department of Mathematical Statistics, Institute of Statistical Studies and Research, Cairo University, who has enriched this work with his helpful suggestions and knowledgeable guidance. It was only through his vast experience, valuable advises, continuous encouragement that this work has been completed and he was the owner of the choice of this thesis. I wish I could he attend this day to see the result of his hard effort for me. So, I hope he will get well soon.

I would like to express my deep appreciation and sincere thanks to **Dr. Fatma Amin, Associate Prof. of Statistics,** Department of Statistics, Faculty of Commerce, Al-Azhar university, Girls' Branch, for her patience and excellent guidance, helpful discussions and continuous encouragements during the developments of this work. Her patience, confidence, insights, encouragement are the light that lightened my way to fulfill this work. Without her continuous support and constructive criticism, this work would never have reached completion.

I am deeply indebted offer my sincerest gratitude to my supervisor **Dr. Hanan M. Aly, Associate Prof. of Statistics,** Department of Statistics, Faculty of Economics and Political Science, Cairo University, for her supported guide throughout my thesis with her patience and knowledge. She provided me intellectual support and resources for the work presented in this thesis. Without her encouragement and effort, this thesis would not have been completed. I feel lucky to have had such encouraging and thoughtful mentor, and I am especially grateful that I am a better person for being her student. I hope I am able to mentor my future students in the way that prof. Hanan has mentored me.

Many thanks are due to the judgment committee **Prof. Badrekan, Z. Mohamed, Prof. of Statistics,** Department of Statistics, Previous Dean of Faculty of Commerce, Al-Azhar university, Girls' Branch, and **prof. Mahmoud, R. Mahmoud, Prof. of Statistics,** Department of Mathematical Statistics, Institute of Statistical Studies and Research, Cairo University, for accepting participating in the discussion and evaluation of this research. With the help of their comments and corrections the research will be in the best proper form.

I am gratefully acknowledging my parents for their love, support and continuous encouragements. I thank my brother, sisters and everyone who asked ALLAH to bring me forth. Finally, thanks to everyone who helped me.

To my parents, to my husband and to my beloved sons

ABSTRACT

With rapidly changing technologies, higher customer expectations for better reliability, need rapid device development and the necessity and the creativeness of more advanced technology in manufacturing field. In the traditional life data analysis, time-to-failure (of a product, system or component) data, obtained under usage conditions are analyzed in order to quantify the life characteristics and to make prediction about products performance. In many situations and for many reasons, it may be difficult or even impossible to obtain such life data. Therefore, in order to observe product failures for analyzing their failure modes and understanding their life characteristics in a short time, accelerated life tests (ALT) are developed.

ALT is usually conducted by subjecting the product (or component) to severe conditions than those the product will be experiencing at normal conditions or by using the product more intensively than in normal use without changing the normal operating conditions. Through ALT, stress levels which accelerate product failure are increased and life data for the product under accelerated stress conditions are captured. Those failure data under accelerated stress conditions are then utilized to derive failure information under usage condition based on some life-stress relationship. Thus, ALT for a product or material is often used to quickly obtain information on the life distribution or product performance under usage conditions.

In ALT data analysis, we need to determine the pdf at normal use conditions from ALT data instead of from regular failure data obtained under normal use conditions. In order to do that, we must have an underlying life distribution and a life-stress relationship.

In this thesis, we assumed failure times at each stress level to be inverse Gaussian (IG) distribution with parameter μ related to the stress through inverse power law model. For constant stress ALT (CSALT), the maximum likelihood estimators of the model parameters, Fisher information matrix, the asymptotic variance-covariance matrix, the confidence bounds, the predictive value of the parameter μ and the reliability function under usual conditions were obtained under both Type-I and Type-II censoring in Chapter 2. Moreover, the optimal design of the ALT was studied according to the *A*-optimality criterion. *A*-optimality criterion for choosing the optimal values of the censored time corresponding to Type-I censoring,

the optimal values of sample sizes at each stress level corresponding to Type-II censoring are described. Also, in case of CSALT, numerical simulation and an illustrative real example are presented for illustration the proposed procedure by considering three stress levels.

Also, Bayesian analysis is presented under Type-II censoring for CSALT in Chapter 3. Markov Chain Monte Carlo (MCMC) simulation algorithm based on Gibbs sampling and WINBUGS software enhance the flexibility of the proposed method developed to estimate the unknown parameters of interest in case of both non-informative and informative priors. Moreover, confidence intervals are constructed and the value of the parameter μ under usual conditions is predicted. Also, in this case, numerical simulation and an illustrative real example are presented for illustrating the proposed procedure by considering three stress levels.

Finally, the statistical inference of ALT under both Type-I and Type-II censoring for simple Step Stress ALT (SSALT) is discussed in Chapter 4. The maximum likelihood estimators of the model parameters, Fisher information matrix, the asymptotic variance-covariance matrix, the confidence bounds, the predictive value of the parameter μ and the reliability function under usual conditions are obtained. Moreover, the optimal design of the ALT is studied according to the *A*-optimality criterion. This criterion is used for determining the optimum time of changing stress value and censored time only under Type-I censoring. Also, in case of SSALT, numerical simulation is introduced to illustrate the proposed procedure by considering two stress levels.

ABBREVIATIONS AND SYMBOLS

ALT	Accelerated life tests
CE	Cumulative exposure
CDF	Cumulative distribution function
CI	Confidence interval
CSALT	Constant stress accelerated life tests
CSPALT	Constant stress partially accelerated life tests
FALT	Fully accelerated life tests
GIG	Generalized inverse Gaussian
GAV	Generalized asymptotic variance
HALT	Highly accelerated life testing
HASS	Highly accelerated stress screening
IG	Inverse Gaussian
LS	Least Squares
LB	Lower bound
MCMC	Markov Chain Monte Carlo
MC error	Monte Carlo standard error
ML	Maximum likelihood
MLSE	Modified least squares estimators
MLEs	Maximum likelihood estimators
MSE	Mean squared errors
PALT	Partially accelerated life testing
Pdf	Probability density function
PH	Proportional hazards
PWM	Probability weighted moment
PSALT	Progressive stress accelerated life tests
RAB	Relative absolute biases
$R(t_0)$	Reliability function
SD	Standard deviation
SSALT	Step stress accelerated life tests
SSPALT	Step stress partially accelerated life tests

ABBREVIATIONS AND SYMBOL (Cont.)

T-H	Temperature-humidity
T-NT	Temperature-non-thermal
UB	Upper bound
WLS	Weighted least squares

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