



Cairo University

# **ELASTIC AND AEROELASTIC ANALYSIS OF AIRCRAFT METALLIC, COMPOSITE, AND SMART WINGS**

By

**Mohamed Abdou Mahran Kasem**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of

**DOCTOR OF PHILOSOPHY**  
in  
**Aerospace Engineering**

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
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METALLIC, COMPOSITE, AND SMART WINGS**

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Aeroelasticity, finite elements, Smart materials, Composites, swept wings

**Summary:**

In the present dissertation elastic and aeroelastic models are developed for the analysis of metallic and composite wings with attached piezoelectric actuators using the finite element method. The elastic and aeroelastic problems were modeled as eigenvalue problems. The piezoelectric potential effect is integrated as prestresses using the concept of geometric stiffness. The analysis covers 2-D and 3-D wings, and the effect of engine mass.

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# **Dedication**

To my Mother Ragaa, my father Abdou, and my wife Eman

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# Abstract

There is an increase in the demand for decreasing the aircraft weight for economic issues. This leads to a decrease in the aircraft stiffness and subsequently the tendency of instability phenomena, such as wing divergence and flutter, to take place. One solution to overcome such problems is the effective use of composite structures, and recently the integration of piezoelectric structures. A novel aeroelastic model for the analysis of metallic and composite wings with attached piezoelectric patches, using the finite element method, is developed. It is based on a 9-node smart composite shell element that is selected based on a comparison among different elements that were found in the literature. The mathematical model is presented and validated in detail.

The model is subsequently used to investigate the effect of piezoelectric forces on the composite wing elastic and aeroelastic modes. Several configurations are considered in the present study. For instance, in composite wings, two wing configurations are considered: one with the piezoelectric sheets at the wing root, while the second model with piezoelectric patches distributed over the wing surface. The elastic analysis shows that the wing twisting mode is significantly improved by the second wing configuration in comparison to the first one. In the aeroelastic analysis, both wing configurations significantly increase the aeroelastic stability of composite wings by increasing both the divergence and flutter speeds. However, the second wing configuration effect is more significant than the first one, especially for divergence analysis. The piezoelectric effect is studied for a voltage range from 0 to 100 volts. Results also show that the piezoelectric effect is more significant in plate-like wings, but it may not be suitable for large scale or 3D wings. The present model can be used effectively to determine the best piezoelectric patches distribution over a composite wing for better elastic and aeroelastic performance, and also to control certain wing modes. It can also be used along with the appropriate optimization schemes to determine the best piezoelectric distribution and applied voltage for better elastic and aeroelastic performance.

The model is also used to investigate the effect of engine mass on the wing elastic and aeroelastic performance. It is found that the best engine mass position is to be at the leading edge between 30% and 40% of the wing span for better elastic and aeroelastic performance.

Experimental investigations show that it is important to investigate the practical limitations of the present numerical analysis as well as the use of piezoelectric materials in real life problems. For instance, the area covered by the piezoelectric sheets should be significant for remarkable stiffening effect. However, the increase of piezoelectric sheets over the plate wing is found to increase the wing mass and, consequently, change its dynamic performance.

# Chapter 1 Introduction

Since the first time piezoelectric materials were introduced, they have been used in numerous structural applications, such as structural beams [1]–[3] and plates [4], [5]. The use of piezoelectric structures either as sensors or actuators was found to be effective for structural analysis [6] and control [4], especially for small scale applications [7]. That is because of the ability of the piezoelectric, as a smart material, to transform mechanical strain into electric displacement and vice versa. For these reasons, piezoelectric materials have been implemented in a wide range of structural and control applications.

Smart materials have been applied to elastic and aeroelastic problems, and in composite and metallic structures. These models have been tested using numerical or experimental methods. The aim of the present work is to develop an aeroelastic model to investigate the metallic, composite, and smart wing elastic and aeroelastic performance using the finite element method. The present section provides a brief introduction to composite structures, smart structures, finite element method, wing aeroelastic analysis as well as a solid foundation for the prospective model.

## 1.1. Composite structures

The need to decrease aircraft weight and cost is the main driver towards introducing new materials in aircraft structures. Therefore, composite materials have become very popular and widely used in aircraft structures for their relatively large strength to weight ratio. In general, a composite material consists of two or more materials that are integrated together on the macroscopic level to end up with a new material with enhanced properties. The composite materials used in aircraft structures usually consist of fibers embedded in a matrix.

### **Fiber reinforcement**

Fibers are usually made of strong and stiff materials such as glass or carbon [8]. Different fiber forms available are twos, whiskers, and nanotubes. One composite material may have one or more fiber types. In aircraft structures composite layers are laminated over each other with different fiber directions to provide appropriate strength and stiffness in the desired directions.

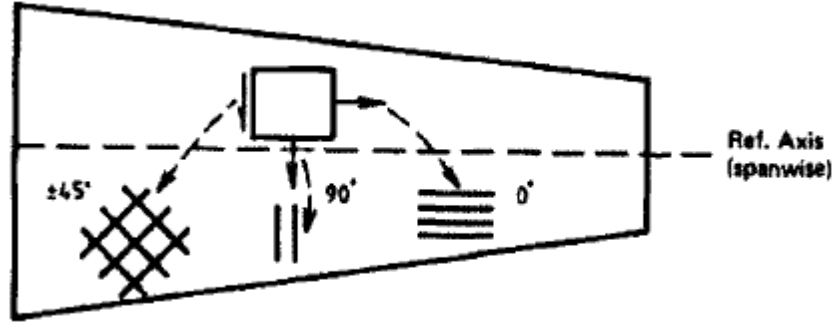


Figure 1-1. Wing composite layup

## Matrices

The matrix provides the medium which protects and bonds the fibers together and transfers load between them.

## 1.2. Piezoelectric materials

Smart materials are known as those materials that have unusual coupling properties [9]. Piezoelectric materials are one type of those. Piezoelectric effect includes the transformation of mechanical displacement into electric signal and vice versa. This special property is the main reason for the piezoelectric material to be engaged in several aerospace applications, such as wing vibration control and de-icing [10]. This section introduces the piezoelectric material and its mathematical modeling.

The piezoelectric material is defined by its Young's modulus, the relative permittivity, voltage output coefficients, coupling coefficients, and piezoelectric voltage coefficients. Examples of piezoelectric materials in industry can be found in [11]. From these properties, one can formulate the piezoelectric relations for structural modeling. The piezoelectric relations for electrical and mechanical coupling can be expressed as [9]

$$\begin{aligned}\boldsymbol{\varepsilon} &= \mathbf{s} \boldsymbol{\sigma} + \mathbf{d} \mathbf{E} \\ \mathbf{D}_e &= \mathbf{d}^T \boldsymbol{\sigma} + \hat{\mathbf{O}}^T \mathbf{E}\end{aligned}\tag{1.1}$$

$\boldsymbol{\varepsilon}$  and  $\boldsymbol{\sigma}$  are the mechanical strain and stress, respectively.  $\mathbf{d}$  is the strain constant matrix,  $\mathbf{D}_e$  is the electric displacement,  $\hat{\mathbf{O}}$  is the dielectric matrix, and  $\mathbf{E}$  is the electric

field vector. The first equation is the actuator equation, while the second one is the sensor equation.

Assuming plane-stress case the elements of equation (1.1) can be expressed as

$$\begin{aligned} \mathbf{s} &= \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_1} & 0 \\ 0 & 0 & \frac{2(1+\nu_{12})}{E_1} \end{bmatrix} \\ d &= \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{32} \\ 0 & 0 & 0 \end{bmatrix} \\ \dot{\mathbf{o}} &= \begin{bmatrix} \dot{o}_{11} & 0 & 0 \\ 0 & \dot{o}_{22} & 0 \\ 0 & 0 & \dot{o}_{33} \end{bmatrix} \end{aligned} \quad (1.2)$$

With some manipulations, the actuator equation can be transformed into another simplified formula for elastic analysis

$$\boldsymbol{\sigma} = \mathbf{Q} \boldsymbol{\varepsilon} - \mathbf{e} \mathbf{E} \quad (1.3)$$

Where

$$\begin{aligned} \mathbf{Q} &= \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \\ \mathbf{e} &= \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & 0 & 0 \end{bmatrix} = \mathbf{d} \mathbf{Q} \end{aligned} \quad (1.4)$$