Ain Shams University

Women's College of

"Arts , Science, and Education"

Mathematics Department

Application of Field Theories in the Domain of Cosmology

A THESIS

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FOR THE DEGREE OF

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BY

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Contents

Abstract

This thesis is concerned with application of field theories in the domain of cosmology. The field theories reviewed are constructed in two different geometries, Riemannian geometry and Absolute Parallelism (AP) geometry. Problems facing the general theory of relativity (GR) and its standard cosmology show clearly that the theory and its underlying geometry should be modified or replaced by other theories and geometries. This is shown clearly in Chapter I of the thesis, terminated by the aim of the work. In Chapter II, we review briefly another type of geometry, the AP-geometry with theories constructing in its domain, including GR itself. We show that such theories suffer from many problems although such theories succeeded in solving some of the GR problems in the domain of cosmology.

A common feature of the theories reviewed in the two Chapters is their material distributions, which are described by a phenomenological tensor (not defined from the building block (BB) of the geometry used). It appears that this may be a reason for problems facing such field theories. In Chapter III, we construct a new field theory in the context of a new version of AP-geometry known as Parameterized Absolute Parallelism (PAP)-geometry. We show that it is a gravity theory with a geometric material distribution (constructed from the BB of the geometry). We also show that the suggested theory can be reduced to GR under a certain condition. The main results of this Chapter is published in an international journal.

Key Words: Cosmology, Modified gravity theories, Parameterized Absolute Parallelism Geometry.

Summary

The thesis contains three Chapters. It contains also a list of abbreviations, symbols, tables and a list of references and an English and Arabic summary. The references in the list are arranged alphabetically and enumerated consequently.

Chapter I: Field Theories in Riemannian Geometry and their Cosmological Applications

> This Chapter reviews briefly the general theory of relativity (GR), the standard theory of gravity. Also, it reviews FRW-standard cosmology. Some problems of GR in the domain of cosmology are reviewed. Also, it contains an alternative theory of gravity, f(R) gravity theory, that is constructed in the context of Riemannian geometry. Moreover, we review a contribution of f(R) to interpret the accelerating expansion of the Universe.

Chapter II: Field Theories in The AP-Geometry and Their Cosmology

> This Chapter, gives a brief account on a more wider geometry than the Riemannian geometry, the AP-geometry. Also we review two theories constructed in this version of geometry. These are the teleparallel equivalence of general relativity

(TEGR) and f(T) theories. Moreover, it contains standard and non-standard cosmological application performed in the context of f(T) theories.

Chapter III: **A Suggested Field Theory in The PAP-Geometry**

In this chapter, we give a brief account on PAP-geometry. Also, we give a new field theory constructed in the context of the PAP-geometry. The field equations are derived using the differential identity method. The theory used is a pure geometric one. We have used three schemes to extracting physical meaning of the geometric objects included in the field theory. A spherical symmetric solution of the field equations of the suggested theory is obtained. This solution gives rise to the Schwarzchild exterior field.

In this thesis Sections are enumerated from 1 to 17, along the thesis, while equations are enumerated after the Section number. Tables and Figures are enumerated after the Chapter number.

List of Abbreviations

AP-Geometry:	Absolute Parallelism geometry
B-B:	Big-Bang
BB:	Building Block
FRW:	Friedmann-Roberston-Walker
GR:	General Relativity
LIGO:	Laser Interferometer Gravitational-wave Observatory
L.H.S.:	Lift Hand Side
R.H.S.:	Right Hand Side
SN:	SuperNouvae
PAP-Geometry:	Parametrized Absolute Parallelism geometry
TEGR:	Teleparallel Equivalent of General Relativity
$:=$ or $\stackrel{\text{def}}{=}$:	Definition

List of Symbols

$g_{\mu\nu}$: Geometric gravitational potential (coefficient of metric)

g: Determinant of the metric of $g_{\mu\nu}$

$\delta^{\mu}{}_{\alpha}$: kroncker delta

η_{ij}: Minkowsky metric

$\{^{\alpha}_{\mu\sigma}\}$: Levi-Civita linear connection

$R^{\alpha}{}_{\mu\nu\sigma}$: Riemannian Curvature tensor

$R_{\mu\nu}$: Ricci tensor

R: Ricci scalar

$G^{\mu}{}_{\alpha}$: Einstein tensor

$T_{\mu\nu}$: Phenomenological material-energy tensor

Λ: Cosmological constant

ds: Metric of space-time(line element)

$a(t)$: Scale factor

k: Sectional Curvature constant

ρ: Proper Energy density

P: Proper Pressure

$\underset{i}{\lambda}_{\mu}$: Tetrad vector

$\Gamma^{\alpha}{}_{\mu\nu}$: Weitzenböck Linear Connection

$\Lambda^{\alpha}{}_{\mu\nu}$: Torsion tensor

$\gamma^{\alpha}{}_{\mu\nu}$: Contortion tensor

$\nabla^{\alpha}{}_{\mu\nu}$: Parameterized Weitzenböck Connection

$\overset{*}{\Lambda}{}^{\alpha}{}_{\mu\nu}$: Parameterized torsion tensor

$\overset{*}{\gamma}{}^{\alpha}{}_{\mu\nu}$: Parameterized Contortion tensor

$\overset{*}{T}_{\mu\nu}$: Geometrical Material-energy tensor

List of Tables

Chapter I

Field Theories in Riemannian Geometry and Their Cosmological Applications