



Ain Shams University

Women's College of

"Arts , Science, and Education"

Mathematics Department

Application of Field Theories in the Domain of Cosmology

A THESIS

SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE (M.Sc.)

(APPLIED MATHEMATICS)

BY

Shymaa Ahmed Ali Refaey

Department of Mathematics

Women's College

Ain Shams University

Supervisors

Prof. Mamdouh I. Wanas

Prof. of Relativistic Cosmology

Astronomy Department

Faculty of science

Cairo University

Prof. Samia S.Elazab

Prof. of Applied Mathematics

Women's College

Ain Shams University

Dr. Samah A. Ammar

Lecturer of Applied Mathematics

Women's Collage –Ain shams university

2018



Ain Shams University

Women's College of

"Arts , Science, and Education"

Mathematics Department

M.Sc.Thesis

(Applied Mathematics)

Title of Thesis:

Application of Field Theories in the Domain of Cosmology

Thesis Supervisors

Prof. Mamdouh I. Wanas

Prof. of Relativistic Cosmology

Astronomy Department

Faculty of science

Cairo University

Prof. Samia S.Elazab

Prof. of Applied Mathematics

Women's College

Ain Shams University

Dr. Samah A. Ammar

Lecturer of Applied Mathematics

Women's Collage –Ain shams university

Approval stamp

/ / 2018

Date of Approval

/ / 2018

Approval of Faculty Council

/ / 2018

Approval of Faculty Council

/ / 2018

Acknowledgment

I would like to express my deep gratitude to Professor M. I. Wanas, Professor Samia S. ELazab and Doctor Samah A. Ammar, for their patient guidance, continuous encouragement and support during their supervision. I am indebted to professor M. I. Wanas for suggesting the point tackled in the thesis.

I also thank members of the Egyptian Relativity Group (ERG). I would also like to extend my thanks to the member of Mathematics Department, Women's College, Ain Shams University, Cairo, Egypt.

Finally, I would like to express my grateful to my Parents and my brothers for supporting me throughout my life in general.

Contents

Abstract	v
Summary	vii
List of Abbreviations	ix
List of Symbols	x
List of Tables	xii
I Field Theories in Riemannian Geometry and Their Cos-	
mological Applications	
1 Introduction	1
2 The General Theory of Relativity	2
2.1 Basic Principles	2
2.2 Field Equations	3
2.3 The Equations of Motion	5
3 Bases of FRW-Standard Cosmology	6
3.1 The Cosmological Principle	6
3.2 Riemannian Structure Satisfying the Cosmological Principle	7
3.3 The FRW-Dynamical Equations	9

3.4	FRW-World Models	12
3.5	Cosmological Parameters	16
4	f(R) Theory and its Cosmological Applications	18
4.1	f(R) Lagrangian Function	19
4.2	Field Equations	20
4.3	f(R) Cosmology	25
5	General Discussion and Aim of the Work	27

II Field Theories in the AP-Geometry and Their Cosmology

6	Introduction	32
7	AP-Geometry	32
7.1	Vanishing Curvature and Non-vanishing Torsion	35
8	Teleparallel Equivalent of General Relativity	37
8.1	Choice of Torsion Scalar	37
8.2	The Field Equations	42
8.3	Conservation Law	46
9	TEGR-Cosmology	46
9.1	The Dynamical Equations	46
9.2	Non-Standard Cosmology	47
10	f(T) Theory and its Cosmology	51
10.1	f(T) Field Equations	52
10.2	f(T) Standard Cosmology	53
10.3	f(T) Non-Standard Cosmology	55
11	Discussion and Criticism	58

III A Suggested Field Theory in the PAP-Geometry

12	Introduction	61
13	A Brief Review of PAP-Geometry	62
14	A Suggested Field Theory	65

14.1	Choice of the Lagrangian Density in PAP-Geometry	65
14.2	The Field Equations	67
14.3	The Equations of Motion	74
15	Extraction of Physics from Geometry	74
15.1	Comparison with Non-linear Field Theories	75
15.2	Linearization Scheme	77
15.3	Type Analysis	83
16	Spherical Symmetry Solution	84
17	Discussion and Concluding Remarks	98
	References	101
	Arabic Summary	109

Abstract

This thesis is concerned with application of field theories in the domain of cosmology. The field theories reviewed are constructed in two different geometries, Riemannian geometry and Absolute Parallelism (AP) geometry. Problems facing the general theory of relativity (GR) and its standard cosmology show clearly that the theory and its underlying geometry should be modified or replaced by other theories and geometries. This is shown clearly in Chapter I of the thesis, terminated by the aim of the work. In Chapter II, we review briefly another type of geometry, the AP-geometry with theories constructing in its domain, including GR itself. We show that such theories suffer from many problems although such theories succeeded in solving some of the GR problems in the domain of cosmology.

A common feature of the theories reviewed in the two Chapters is their material distributions, which are described by a phenomenological tensor (not defined from the building block (BB) of the geometry used). It appears that this may be a reason for problems facing such field theories. In Chapter III, we construct a new field theory in the context of a new version of AP-geometry known as Parameterized Absolute Parallelism (PAP)-geometry. We show that it is a gravity theory with a geometric material distribution (constructed from the BB of the geometry). We also show that the suggested theory can be reduced to GR under a certain condition. The main results of this Chapter is published in an international journal.

Key Words: Cosmology, Modified gravity theories, Parameterized Absolute Parallelism Geometry.

Summary

The thesis contains three Chapters. It contains also a list of abbreviations, symbols, tables and a list of references and an English and Arabic summary. The references in the list are arranged alphabetically and enumerated consequently.

Chapter I: Field Theories in Riemannian Geometry and their Cosmological Applications

This Chapter reviews briefly the general theory of relativity (GR), the standard theory of gravity. Also, it reviews FRW-standard cosmology. Some problems of GR in the domain of cosmology are reviewed. Also, it contains an alternative theory of gravity, $f(R)$ gravity theory, that is constructed in the context of Riemannian geometry. Moreover, we review a contribution of $f(R)$ to interpret the accelerating expansion of the Universe.

Chapter II: Field Theories in The AP-Geometry and Their Cosmology

This Chapter, gives a brief account on a more wider geometry than the Riemannian geometry, the AP-geometry. Also we review two theories constructed in this version of geometry. These are the teleparallel equivalence of general relativity

(TEGR) and $f(T)$ theories. Moreover, it contains standard and non-standard cosmological application performed in the context of $f(T)$ theories.

Chapter III: A Suggested Field Theory in The PAP-Geometry

In this chapter, we give a brief account on PAP-geometry. Also, we give a new field theory constructed in the context of the PAP-geometry. The field equations are derived using the differential identity method. The theory used is a pure geometric one. We have used three schemes to extracting physical meaning of the geometric objects included in the field theory. A spherical symmetric solution of the field equations of the suggested theory is obtained. This solution gives rise to the Schwarzschild exterior field.

In this thesis Sections are enumerated from 1 to 17, along the thesis, while equations are enumerated after the Section number. Tables and Figures are enumerated after the Chapter number.

List of Abbreviations

AP-Geometry:	Absolute Parallelism geometry
B-B:	Big-Bang
BB:	Building Block
FRW:	Friedmann-Roberston-Walker
GR:	General Relativity
LIGO:	Laser Interferometer Gravitational-wave Observatory
L.H.S.:	Lift Hand Side
R.H.S.:	Right Hand Side
SN:	SuperNouvae
PAP-Geometry:	Parametrized Absolute Parallelism geometry
TEGR:	Teleparallel Equivalent of General Relativity
$:=$ or $\stackrel{\text{def}}{=}$:	Definition

List of Symbols

$g_{\mu\nu}$:	Geometric gravitational potential (coefficient of metric)
g :	Determinant of the metric of $g_{\mu\nu}$
δ^μ_α :	kroncker delta
η_{ij} :	Minkowsky metric
$\{\mu\sigma\}^\alpha$:	Levi-Civita linear connection
$R^\alpha_{\mu\nu\sigma}$:	Riemannian Curvature tensor
$R_{\mu\nu}$:	Ricci tensor
R :	Ricci scalar
G^μ_α :	Einstein tensor
$T_{\mu\nu}$:	Phenomenological material-energy tensor
Λ :	Cosmological constant
ds :	Metric of space-time(line element)
$a(t)$:	Scale factor
k :	Sectional Curvature constant
ρ :	Proper Energy density
P :	Proper Pressure
λ_μ :	Tetrad vector
$\Gamma^\alpha_{\mu\nu}$:	Weitzenböck Linear Connection
$\Lambda^\alpha_{\mu\nu}$:	Torsion tensor
$\gamma^\alpha_{\mu\nu}$:	Contortion tensor
$\nabla^\alpha_{\mu\nu}$:	Parameterized Weitzenböck Connection

$\overset{*}{\Lambda}{}^\alpha{}_{\mu\nu}$:	Parameterized torsion tensor
$\overset{*}{\gamma}{}^\alpha{}_{\mu\nu}$:	Parameterized Contortion tensor
$\overset{*}{T}{}_{\mu\nu}$:	Geometrical Material-energy tensor

List of Tables

II.1 Second Order World Tensors [27].....	40
III.1 Expansion of the geometric objects [54].....	87
III.2 Type Analysis.....	91

Chapter I

Field Theories in Riemannian Geometry and Their Cosmological Applications