*Faculty of Education*
*Department of Mathematics*

# On Study of Nonlinear Partial Differential Equations in Mathematical Physics

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# SUMMARY

The objective of this thesis is to study the nonlinear partial differential equations (PDEs) in mathematical physics in order to find new exact solutions including Jacobi elliptic function solutions, hyperbolic function solutions, trigonometric function solutions, rational function solutions, solitons and other solutions for them by applying various methods. These methods are: the new extended auxiliary equation method, the new mapping method, the Jacobi elliptic equation method, the generalized Riccati equation mapping method, the generalized Kudryashov method, the $\phi^6$-model expansion method, the first integral method, the rational $(G'/G)$-expansion method, the generalized sub-ODE method, the soliton ansatz method, the sine-cosine method, the Riccati equation expansion method, the $(G'/G)$-expansion method, the Bernoulli equation method, the modified simple equation method, the exp-function method, the general $\text{Exp}_a$-function method, the multiple exp-function method and the linear superposition principle.

This thesis consists of seven chapters, together with an Arabic and English summaries. It is organized as follows:

**In chapter one**, we comprise an introduction contained in a brief survey. Development and survey of the available mathematical tools used to generate exact solutions for nonlinear PDEs given in chapters II-VII.

**In chapter two**, we apply four different methods to find many new exact solutions of eight nonlinear PDEs. Firstly in sections 2.1-2.3, we apply a new extended auxiliary equation method to find many new Jacobi elliptic function solutions, hyperbolic function solutions and trigonometric function solutions of the higher-order nonlinear Schrödinger equation with derivative non-Kerr nonlinear terms, the higher-order

dispersive nonlinear Schrödinger equation, and the generalized nonlinear Schrödinger equation. Secondly in sections 2.4-2.6, we apply a new mapping method to find solitons and other exact solutions of the $(2+1)$-dimensional nonlinear cubic-quintic Ginzburg-Landau equation, the $(1+1)$-dimensional resonant nonlinear Schrödinger's equation with parabolic law nonlinearity, and the $(1+1)$-dimensional nonlinear generalized Zakharov system. Thirdly in section 2.7, we apply the Jacobi elliptic equation method to find Jacobi elliptic function solutions, hyperbolic function solutions and trigonometric function solutions of the perturbed nonlinear Schrödinger equation, with power law nonlinearity and Hamiltonian perturbed terms. Fourthly, in section 2.8, we use the generalized Riccati equation mapping method to solve the nonlinear Bogoyavlenskii equations and many new exact solutions are obtained.

**In chapter three**, we use the generalized Kudryashov method for finding exact solutions of seven nonlinear PDEs namely, the nonlinear Burgers equation , the modified Benjamin-Bona-Mahony (mBBM) equation, the Cahn-Hilliard equation, the diffusive predator-prey system, the nonlinear telegraph equation, the Biswas-Milovic equation with dual-power law nonlinearity, and the Zakharov-Kuznetsov equation (ZK(m,n,k)). As a result, many analytical exact solutions are obtained including symmetrical Fibonacci function solutions and hyperbolic function solutions.

**In chapter four**, we apply four different methods to find solitons and other solutions of three nonlinear PDEs. Firstly, in section 4.1, we apply the $\phi^6$-model expansion method to find new exact solutions of the higher-order nonlinear Schrödinger equation with derivative non-Kerr nonlinear terms. Secondly in sections 4.2 and 4.3, we apply the first integral method and the rational $(G'/G)$-expansion method for finding the exact solutions of the Biswas-Milovic equation with dual-power law nonlinearity. Thirdly, in sections 4.4 and 4.5, we apply the generalized sub-ODE method and the rational $(G'/G)$-expansion method for finding exact solutions of the GKdV-mKdV equation with higher-order nonlinear terms.

**In chapter five**, we apply two different methods to find bright-dark-singular soliton solutions and trigonometric function solutions of eight nonlinear PDEs. Firstly, we apply the soliton ansatz method to find the bright-dark-singular soliton solutions for six nonlinear PDEs namely, the perturbed nonlinear Schrödinger equation with Kerr law nonlinearity, the higher-order nonlinear Schrödinger equation, the generalized nonlinear Schrödinger equation with a source, the long-short wave resonance equations, the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation and the nonlinear foam drainage equation.

Secondly, we apply the sine-cosine method to find the trigonometric function solution for the perturbed nonlinear Schrödinger equation, with power law nonlinearity and Hamiltonian perturbed terms and, the GKdV-mKdV equation with higher-order nonlinear terms.

**In chapter six**, we apply the soliton ansatz method combined with other methods namely: the Jacobi elliptic equation method, the Riccati equation expansion method, the $(G'/G)$-expansion method, the Bernoulli equation method, the extended auxiliary equation method, the new mapping method and the $\phi^6$-model expansion method for solving the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation, the nonlinear foam drainage equation, the nonlinear Schrödinger equation with fourth-order dispersion and cubic-quintic nonlinearity, and a variant nonlinear Boussinesq equations.

**In chapter seven**, we apply four different techniques to find the exact traveling wave solutions for some nonlinear PDEs. Firstly, we will use the modified simple equation method for the generalized nonlinear Schrödinger equation with a source, the long-short wave resonance equations, the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation, the nonlinear foam drainage equation and a variant nonlinear Boussinesq equations. Secondly, we will use the exp-function method for the GKdV-mKdV equation with higher-order nonlinear terms, the long-short wave resonance equations, the (2+1)-dimensional hyperbolic nonlinear Schrödinger equation, the nonlinear foam drainage equation. Thirdly, we will use the general $\text{Exp}_a$-function method for the nonlinear Schrödinger equation with fourth-order dispersion and cubic-quintic nonlinearity, and a variant nonlinear Boussinesq equations. Fourthly, we will use the multiple exp-function method and the linear superposition principle for the following (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff (CBS) equation.

All the results of this thesis have been published in internationally refereed scientific journals as follows:

1. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, New extended auxiliary equation method for finding many new Jacobi elliptic function solutions of three nonlinear Schrödinger equations, Waves in Random and Complex Media, 27 (2017) 420-439.[76] (2016/2017 Impact Factor: 1.447)

2. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, Solitons and other exact solutions for a class of nonlinear Schrödinger -type equations, Optik- International Journal for Light and Electron Optics, 130 (2017) 1295-1311.[87] (2016/2017 Impact Factor: 0.835)

3. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, Exact solutions for the perturbed nonlinear Schrödinger equation with power law nonlinearity and Hamiltonian perturbed terms, Optik, 139 (2017) 123-144.[81] (2016/2017 Impact Factor: 0.835)

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9. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, Exact solutions and optical soliton solutions of the nonlinear Biswas-Milovic equation with dual-power law nonlinearity, Acta Physica Polonica A, 131 (2017) 240-251.[79] (2016/2017 Impact Factor: 0.469)

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12. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, The modified simple equation method and the method of soliton ansatz for solving the generalized nonlinear Schrödinger equation with a source, Journal of Advanced Mathematical Studies, 9 (2016) 235-244.[91]

13. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, The modified simple equation method, the exp-function method, and the method of soliton ansatz for solving the long–short wave resonance equations, Zeitschrift für Naturforschung A, 71a (2016) 103-112.[92] (2016/2017 Impact Factor: 1.432)

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18. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, Solitons and other exact solutions for variant nonlinear Boussinesq equations, Optik, 139 (2017) 166-177.[89] (2016/2017 Impact Factor: 0.835)

19. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, Generalized Kudryashov method and general $\text{Exp}_a$-function method for solving a higher order nonlinear Schrödinger equation, Journal of Space Exploration, 6 (2017) 1-26 [84]

20. E. M. E. Zayed, Abdul-Ghani Al-Nowehy, The multiple exp-function method and the linear superposition principle for solving the (2+ 1)-dimensional Calogero-Bogoyavlenskii-Schiff equation, Zeitschrift für Naturforschung A, 70a, (2015) 775-779.[93] (2015 Impact Factor: 0.886)

A comparison between our recent results and the well-known results are given. Finally, our results in this thesis have been checked with the aid of the Maple by putting them back into the original equations.