Ain Shams University

Faculty of Education

Department of Mathematics

Study of some problems on the motion of the Newtonian and non-Newtonian Nanofluids.

A THESIS

Submitted in Partial Fulfillment for the Requirement of the Master Degree in Teacher Preparation in Science (Applied Mathematics)

To

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List of publication

1-MHD boundary layer chemical reacting flow with heat transfer of Eyring-Powell Nano fluid past a stretching sheet**.** It is worthwhile to remark that this work has been accepted for publication in the "Journal of Microsystem Technologies", https://doi.org/10.1007/s00542-018-3915-1.

Contents

CONTENTS

Summary

Summary

The aim of this thesis is to investigate some problems on the motion of the Newtoninan and non-Newtonian fluids. Three problems are discussed here.

1. MHD boundary layer chemical reacting flow with heat transfer of Eyring-Powell Nano fluid past a stretching sheet
2. MHD boundary layer Nano-Prandtl fluid flow past a Stretching sheet with heat and mass transfer
3. MHD boundary layer flow with heat and mass transfer of Oldroyed-B Nano fluid past a Stretching sheet through porous medium.

The thesis consists of four chapters:

In chapter 1,

This chapter is a presentation for the following topics:

1.1 Nanofluids.
1.2 The Rheological behavior of fluids.
1.3 Classification of Non-Newtonian fluid.
1.4 Constitutive equations.
1.5 Heat transfer.
1.6 Mass transfer.
1.7 Magneto-hydrodynamics(MHD)
1.8 Boundary layer
1.9 Basic equations of a Newtonian or non-Newtonian fluid.
1.10 Runge Kutta Fehlberg method (RKF45)

In chapter 2,

In this chapter, we study the problem of laminar Nano non-Newtonian fluid flow through the boundary layer, which results from the stretching of a flat surface. The model of Eyring-Powell is used for the fluid. Constant normal magnetic field, mixed convection, chemical reaction, viscous dissipation, ohmic dissipation, Brownian and thermophoresis effect are considered. The problem is modulated mathematically by a system of partial differential equations, which describe the motion. A similarity solution is presented to transform this system to ordinary non–linear differential equations. The numerical solutions of these equations are obtained as functions of the physical parameters of the problem. Such as, Brownian number, thermophoresis number, Lewis number, Prandtl number, Magnetic parameter and Elastic parameter. Graphical evaluation is displayed to depict the intrinsic behavior of embedded parameters on velocity, temperature, and Nano particle concentration profiles. **It is worthwhile to remark that this work has been accepted for publication in the "Journal of microsystem technology".**

In chapter 3,

In this chapter, we have investigated the problem of laminar Nano non-Newtonian fluid flow through a boundary layer, which results from the stretching of a vertical flat surface. The model of Prandtl fluid is used for the flow. The system is stressed by a uniform normal magnetic field. The mixed

convection, Chemical reaction, Ohmic dissipation, Couple stresses, Brownian motion and thermophoresis are considered. The problem is modulated mathematically by a system of nonlinear partial differential equations which describe the motion. A similarity solution is presented to transform this system to ordinary non-linear differential equations. The numerical solutions of these equations are obtained as functions of the physical parameters of the problem. such as, Brownian motion number, thermophoresis number, Lewis number, Prandtl number, Magnetic parameter, mixed convection parameter, couple stress parameter, Eckert number, chemical reaction parameter , Elastic parameter, Nano Lewis number, modified Dufour parameter , and Dufour solutal Lewis number. Local Nusselt number, Sherwood number and Nano Sherwood number are discussed through graphs. Graphical evaluation is displayed to depict the intrinsic behavior of embedded parameters on velocity, temperature, solutal concentration, and Nano particle concentration profiles.

In chapter 4,

In this chapter an analysis of the problem of laminar Nano non-Newtonian fluid flow through the boundary layer which results from the stretching of a flat surface. The model of Oldroyed-B fluid model is used for the fluid. The system is stretched by a uniform normal magnetic field, mixed convection; Chemical reaction, Ohmic dissipation,flow heat generation/absorption, Brownian motion and thermophoresis are considered. The problem is modulated mathematically by a system of nonlinear partial differential equations which describe the motion. A similarity solution is presented to transform this system to ordinary non-linear differential equations. The system of these equations are solved numerically by using shooting method with Runge-Kutta fehlberg technique and The numerical solutions of these equations are obtained as a function of the physical parameters of the problem. such as, Brownian motion number, thermophoresis number, Darcy number, Lewis number, Prandtl number, Magnetic parameter, mixed convection parameter, heat generation/absorption parameter, Eckert number, chemical reaction parameter, Deborah number, Nano Lewis number , modified Dufour parameter, and Dufour solutal Lewis number. Graphical evaluation is displayed to depict the intrinsic behavior of embedded parameters on velocity, temperature, solutal concentration, and Nano particle concentration profiles.

Chapter1
Introduction

Chapter 1

Introduction

1.1 Nano fluids:

1.1.1 Definition of Nano fluids:

The term Nano fluid is used for the fluids having suspension of Nano-metallicparticles. The main idea of using nanoparticle is to enhance the thermal properties of a base fluid. Invokement of Nano fluids with improved heat distinctiveness can be noteworthy in stipulations of more competent cooling system, consequential in higher productivity and energy savings. Several prospective applications for Nano fluids are heat exchangers, radiators for engines, process cooling systems, microelectronics, etc. Choi [1] was the first who have made the analysis on nanoparticles in 1995.

1) **The base fluid:**

- Metallic liquid: sodium (Na).
- Non-metallic liquids: water (H_2O), ethylene glycol (EG), engine oil (EO).

2) **The nanoparticles:**

- Metallic compounds: copper (Cu), Silver (Ag).
- Ceramic compounds:

1. Oxide: alumina (Al_2O_3), copper oxide (CuO), zinc oxide (ZnO).
2. Carbides: Silicon carbide (SIC).
3. Sulfides: copper sulfides (CuS).

- Carbon based compounds: Carbon nanotubes (CNTs), single wall (SWCNT), double wall (DWCNT), multi wall(MWCNT)

1.1.2 Advantages of Nano fluid:

Due to the features of the nanoparticles, Nano fluids have many advantages such as:[2]

1. Nanoparticles have less momentum and hence low kinetic energy leading a reduction in surface erosion.
2. Nano fluids can be used in micro-channels, as nanoparticles do not cause any clogging.
3. For small sizes of nanoparticles, Nano fluids behave like pure fluids as they keep the fluid properties of the base fluid. Further, because the size of nanoparticles is similar to bio-molecules, Nano fluids can be used in biomedical applications.
4. Nano fluids are more stable; as nanoparticles have large ratio of surface area to volume, because of the large number of atoms on the boundaries, making the interaction between the particle surface and the fluid strong enough to overcome the variance in density. `

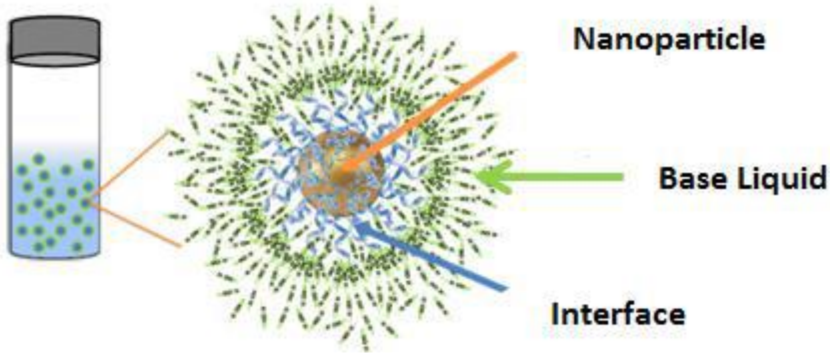

Figure1. Schematic of nanoparticles dispersion systems (NFs) as a three-phase suspension system.

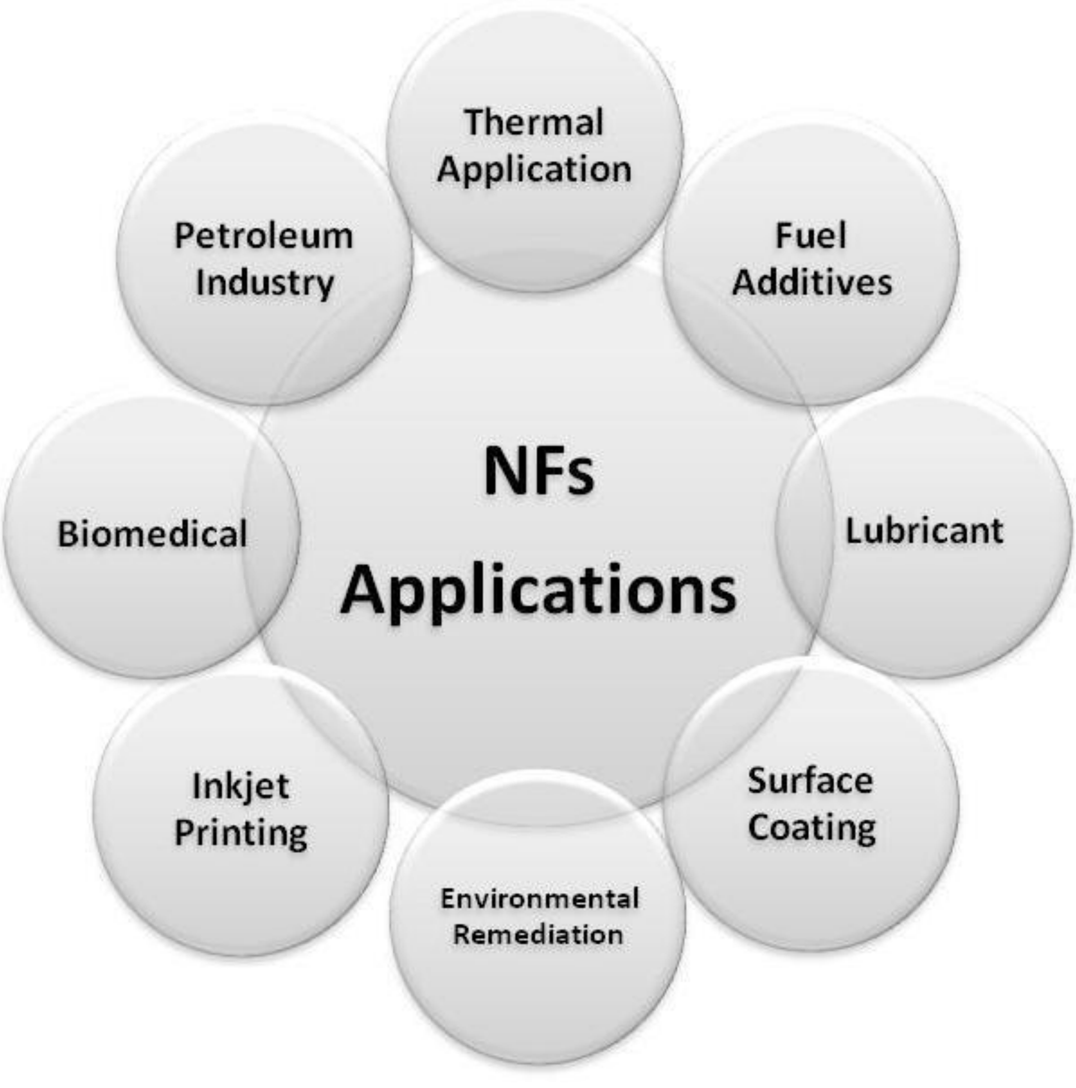

Figure 2. Different applications of nanofluids

1.1.3 Governing equation for Nano fluids:

Buongiorno [3] constructed a transport model for Nano fluids with some assumptions. These assumptions are incompressibility of the flow, no chemical reaction and radiative heat transfer, negligible external forces; dilute mixture (volumetric nanoparticle fraction$\ll 1$) , negligible viscous dissipation, nanoparticles and base fluid are locally in thermal equilibrium. He considered that in the absence of the turbulent effect, only Brownian diffusion and thermophoresis are the two effective slip mechanisms. This can be defined as follows:

- Brownian diffusion coefficient describes Brownian motion that results from continuous collisions between the nanoparticles and molecules of the base fluid. From Einstein-Stokes equation $D_B = \frac{K_B\,T}{3\pi\,\mu\,d_p}$, where d_p is nanoparticle diameter. The nanoparticle mass flux due to Brownian diffusion $J_{P,B} = -\rho_P D_B \nabla\phi$, where ρ_P is the density of the nanoparticle, ϕ is the nanoparticles volume fraction.
- Thermophoresis is the phenomenon occurs when particles diffuse under the effect of temperature gradient. Thethermophoresis diffusion coefficient $D_T = \frac{\alpha\,\mu}{\rho_f}\,\phi$, where α is the thermal proportionality, ρ_f is the fluid density. The nanoparticles mass flux due to themophoresis diffusion is defined as $J_{P,T} = -\rho_P D_T \frac{\nabla T}{T_m}$, where T_m is the mean temperature.

Hence, the sum of the mass flux due to Brownian diffusion and thermophoresis can be represented as follows

$$J_P = J_{P,B} + J_{P,T} = -\rho_P D_B \nabla\phi - \rho_P D_T \frac{\nabla T}{T_m} \tag{1.1}$$

The continuity equation for the nanoparticles, in the absence of the chemical reactions, can be written as:

$$\left(\frac{\partial}{\partial t} + V.\nabla\right)\phi = \frac{-1}{\rho_P}\nabla.J_P = D_B\nabla^2\phi + \frac{D_T}{T_m}\nabla^2 T \tag{1.2}$$

The expression $(V.\nabla)$ ϕ indicate that the nanoparticle can move homogenously within the fluid. The expression $\left(\frac{-1}{\rho_p}\nabla.j_p\right)$ is the diffusion of mass flux for the nanoparticle. This diffusion is the sum of two terms; the first term $(D_B\nabla^2\phi)$ is due to Brownian diffusion and the second term $\left(\frac{D_T}{T_m}\nabla^2 T\right)$ is due to thermophoretic effect.

1.1.4 Application for Nano fluids:

According to "Health Canada" any material can be considered to be a nanomaterial whether it is a manufactured, ingredient, structure or device if one of following two conditions is satisfied [3].

1. It is within the nanostructures in at least one external dimension, or surface structure in Nano scale.
2. It is smaller or larger than Nano scale on all dimensions and displays one or more Nano scale properties.

From this point of view, there are some applications of Nano fluids in food industry:

- In some battery devices like in phones, cars, and laptops. The reason here properties of Nano fluids can be changed by applying an electric field or magnetic field in the so-called smart fluids