



AIN SHAMS UNIVERSITY
DEPARTMENT OF MATHIMATIC
AL STATISTICS



ON MARSHALL – OLKIN TECHNIQUE OF EXTENDING UNIVARITE DISTRIBUTIONS

A Thesis

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Submitted by

WESAL EMHEMED RAMADAN AGHEL

Mathematic Department, Faculty of Science, Alzzawia
University (2009)

Under the Supervision of

Prof. Dr. Mohamed Gharib Mahmoud

Professor of Mathematical Statistics, Department of
Mathematics Faculty of Science, Ain Shams University

Dr . Bahady Ibrahim Mohamed

Lecturer of Mathematical Statistics, Mathematics
Department, Faculty Of Science, Al-Azhar University (men)

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Keywords

The Marshal- Olkin extended inverse pareto distribution,
Exponentiated Marshall-Olkin discrete uniform distribution
,reliability analysis ,maximum likelihood estimation.

ENGLISH SUMMARY

English Summary

problems of establishing and extending new classes and families of discrete and continuous probability distributions are one of the most important research topics in the theory of distributions.

The Kotz and Vicari (2005) gave highlights the most important early developments of statistical distributions. Since 1980, research methods have begun to generate new distributions that tend to add new parameters to known statistical distributions.

This is probably due to the mathematical and analytic capabilities available in software such as **R** (packages), Matlabm, Mable and Mathemahca, through which researchers can easily tackle problems with incomplete beta and gamma functions in generalized family.

The second reason lies in the characteristics of the curved tail of the new statistical distribution produced by adding one or more parameters to old distribution.

Thirdly, this parameters (**s**) induction has also proved to be helpful In improving the goodness of fit of the proposed family of distribution.

In (1997) Marshall and Olkin proposed a technique for adding a parameter to an existing distribution. Marshall and Olkin extended distributions offer a wide range of behavior than the basic distributions form which the are derived. The property that the extended form of distributions can have an testing hazard function depending on the value of the added parameter and therefore can be used to model real situation in a better manner than the basic distribution, resulted in

the detailed of Marshall and Olkin extended family of distribution.

The thesis consists of three chapters as follows:

Chapter I: presents basic concepts and characteristics of Marshall and Olkin distribution and some previous researches related to it. This chapter also gives a brief overview of new distributions generated by this method. The chapter and exposing with some generalizations of Marshall and Olkin generated of distributions.

Chapter II: This chapter introduces a new application of Marshall and Olin method to Inverse Pareto distribution. The statistical properties of the new model are discussed and maximum likelihood used to estimate parameters, quantile function, moments and order statistics. Finally, usefulness of the new model for modeling reliability data is illustrated using two real data sets with simulation study.

The results of this chapter were published in the periodical:
International Journal of Statistics and Probability.2017, 6(6), 71-84.

Chapter III: This chapter presents a new generalization of the Marshall and Olkin method presented by Sandhya and Prasanth (2014). We obtained a new distribution (EMOU) as an application for the proposed distribution. We studied some statistical properties of new model for example hazard rate function, quantile function and moments, then we numerically calculated the mean, the standard deviation, and Shannon's entropy of the given model at different values of parameters. To examine the performance of our new model in fitting several data we

use real set of data to compare the fitting of new model with some well-known models, which provides best fit to all of data.

The results of this chapter were published in the following periodical:

International Journal of Modern Engineering Research
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