



AIN SHAMS UNIVERSITY

FACULTY OF ENGINEERING

Engineering Physics and Mathematics Department

Stability and Control of an Inverted Pendulum Motion

A Thesis submitted in partial fulfillment of the requirements of the degree of

Master of Science in Engineering Mechanics

By

Mohamed Magdy Mohamed Abdo

Bachelor of Science in Electrical Engineering

Faculty of Engineering, Ain Shams University, 2014

Supervised By

Prof. Dr. / Abdallah Mostafa El Marhomy

Dr. / Mahmoud Abd-Allah Attia

Cairo - (2018)



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Statement

This thesis is submitted as a partial fulfillment for the degree of Master of Science in Engineering Mechanics at Faculty of Engineering, Ain Shams University.

The author carried out the work included in this thesis, and no part of it has been submitted for a degree or a qualification at any other scientific entity.

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Publications

1. Mohamed Magdy, Abdallah El Marhomy, Mahmoud A. Attia, “Comparison Between DC motor, Universal Motor and Switched Reluctance Motor Performance on PID Controlled Inverted Pendulum System Tuned by Hybrid GSA-GA Algorithm,” in *4th International Conference of Engineering Division(May 6-8)*, National Research Centre (NRC), Cairo, 2018.
2. Mohamed Magdy, Abdallah El Marhomy, Mahmoud A. Attia, “Modeling of Inverted Pendulum System with Gravitational Search Algorithm Optimized Controller”, under publication at ASEJ, ELSEVIER.

Thesis Summary

An inverted pendulum is a classic control problem. Due to the importance of its model, many researchers had challenged to get the proper controller for this model. The inverted pendulum system under consideration consists of a rigid rod, movable cart, a motor to drive the cart, and a controller. The governing equations of motion of the system have been derived using D'Alembert's principle, and Lagrange's equations.

After studies and investigations on the different optimization techniques, it has been proved that the Hybrid GA-GSA is the most proper one in tuning the controllers and in getting a suitable parameter to decrease the system overshoot and settling time.

DC armature controlled motor, universal motor, and switched reluctance motor have been used to drive the system cart. It has been demonstrated that DC armature controlled motor is the most suitable motor for this task, as it has no oscillations within the urban time. PID and PIDA controllers have been used to control the pendulum angle and to move the cart to the desired position.

At the end of this thesis, we have reached to the conclusion that the DC armature controlled motor with PIDA-Hybrid GA-GSA optimized controller IP system yields the best performance to control and stabilize the motion of the IP.

Keywords:

Inverted Pendulum, Motors, PID controller, PIDA controller, Genetic Algorithm, Gravitational Search Algorithm, D'Alembert's principle, Lagrange's equations.

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