Ain Shams University
Faculty of Science
Mathematics Department

A Study on Interval Estimation Problems in the Stress-Strength Model

A Thesis

Submitted for Ph. D. Degree of Philosophy in Science
in
Mathematical Statistics

By

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Submitted to
Mathematics Department
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Ain Shams University
Cairo Egypt

2018

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موافقة مجلس الكلية **موافقة مجلس الجامعة**

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To spirit of my beloved father.

To my best friend Noha

Acknowledgements

All praise and thanks to **ALLAH** who guided and helped me to bring this thesis.

I am indebted and very grateful to my supervisor **Prof. Dr. Nahed Abd El-Salam Mokhlis**, for her continuous help, guidance, encouragement, permanent support, precise advices, and the amount of time her spent with me in discussing the results. The discussions with her were always fruitful. Without her guidance and encouragement, this thesis would not have been possible.

I would like to express my deep gratitude to my supervisor **Dr. Emad Jacoob Ibrahim**, for his continuous help, support, guidance and encouragement to me.

I would like to express my great thanks to **Prof. Dr. Ahmed Mohamed Rashad**, for his kind encouragement.

Great thanks, to **Prof. Dr. Essam Ahmed Soliman**, Head of Mathematics Department, Faculty of Science, Ain Shams University, for his continuous encouragement and kind help.

Also, I would like to express my sincere appreciation to the staff members of the Mathematics Department, Faculty of Science, Ain Shams University, for their encouragement and amicability.

I will always be grateful to my father, mother, brother, sisters, and friends. I express my since gratitude to their, for their care, great encouragement, and constant support.

Abbreviations

ACI	Approximate confidence interval
BCI	Bayesian credible interval
boot	Bootstrap confidence interval
CDF	Cumulative distribution function
G-BCI	BCI of Gamma priors
GCI	Generalized confidence interval
GEF	General exponential form
GIEF	General inverse exponential form
GPQ	Generalized pivotal quantity
GV	Generalized variable
M-BCI	BCI of mixed priors
MCMC	Markov chain Monte Carlo
MLE	Maximum likelihood estimator
P-boot	Percentile bootstrap confidence interval
PDF	Probability density function
PQ	Pivotal quantity
T-boot	T-bootstrap confidence interval
SF	Survival distribution function

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1. Mokhlis, N., Ibrahim, E. and Gharieb, D. (2017). Stress-strength reliability with general form distributions. *Communications in Statistics-Theory and Methods*, **46**, 1230-1246.
2. Mokhlis, N., Ibrahim, E. and Gharieb, D. (2017). Interval estimation of a $P(X_1 < X_2)$ model with general form distributions for unknown parameters. *Statistics Applications and Probability*, **6**, 391-400.
3. Mokhlis, N., Ibrahim, E. and Gharieb, D. (2018). Simple confidence limits for a $P(X_1 < X_2)$ model involving general forms of distributions. *Far East Journal of Theoretical Statistics*, **54**, 175-188.
4. Mokhlis, N., Ibrahim, E. and Gharieb, D. (2017). Interval estimation of stress-strength reliability for a general exponential form distribution with different unknown parameters. *International Journal of Statistics and Probability*, **6**, 60-70.
5. Mokhlis, N., Ibrahim, E. and Gharieb, D. (2018). Interval estimation of a $P(X_1 < X_2)$ model for variables having general inverse exponential form distributions with unknown parameters. *American Journal of Theoretical and Applied Statistics*, **7**, 132-138.

Abstract

Dina Mahmoud Gharieb Mohamed.

A Study on Interval Estimation Problems in the Stress-Strength Model.

Degree of Ph.D. of Philosophy in Science in Mathematical Statistics.

Mathematics Department, Faculty of Science, Ain Shams University, 2018.

The main objective of this thesis is the interval estimation of a stress-strength model with independent variables possessing distributions with either general exponential form or general inverse exponential form. Different interval estimators are obtained by different methods and under different assumptions. The interval estimators obtained are exact, generalized, approximate, bootstrap, and Bayesian. Various statistical distributions in the literature possess the underlying forms of distributions. So, the results obtained are consistent with many results in the literature, and may be applied to distributions possessing these forms not yet studied. As illustration of the results obtained, the Weibull and inverse Weibull distributions are applied as examples of the two general forms. Simulation studies are also carried out for comparison of the different interval estimators obtained. The comparison is based on average length, average coverage probability, and tail errors.

Keywords: General exponential form, General inverse exponential form, Stress-strength reliability, Interval estimation, Maximum likelihood estimation, Generalized pivotal quantity, Generalized variable, Markov chain Monte Carlo method, Approximate confidence interval, Generalized confidence interval, Bootstrap confidence interval, Bayesian credible interval.

Summary

The terminology stress-strength model "$P(X_1 < X_2)$" makes explicit that both stress, X_1, and strength, X_2, are treated as random variables. Nowadays the stress-strength model is of substantial interest and usefulness in various areas of science such as engineering, reliability theory, psychology, genetics and, clinical trials. In the simplest stress-strength model, X_1 is the stress imposed on the unit by the operating environment and X_2 is the strength of the unit. A unit is able to perform its intended function if its strength is greater than the stress imposed upon it. Reliability is defined as the probability of non-failing, therefore, $R = P(X_1 < X_2)$ is the stress-strength reliability. The research in this area has been conducted all over the world, and the results have appeared in many applications, such as medicine, biostatistics, genetics, industry, engineering, psychology, and quality control. Because of the importance of what we talk about there are a lot of research, which took this topic in different ways with different assumptions. One of these ways is the estimation; the estimation refers to the process by which one makes inferences about a population, based on information obtained from a sample. The estimation of stress-strength model has been widely used in the fields of aeronautical, civil, mechanical and electronic engineering. It is one of the most important issues in statistical inference. The estimation can be expressed in two ways, point and interval estimation. In point estimation, a statistic used to estimate a parameter is called a point estimator. But knowing the point value is not enough, we also want to know how close to the truth it is. In interval estimation, we make statements that the true parameter locates within some region typically depending on the point estimate with some prescribed probability. An