



A NOVEL GENERIC HYBRID ARCHITECTURE FOR RADIX-R FAST FOURIER TRANSFORM

By

Mahmoud Ismail Soliman Nazmy Elbeltagy

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfilment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Electronics and Communications Engineering

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Title of Thesis:

A NOVEL GENERIC HYBRID ARCHITECTURE FOR
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Key Words:

Fast Fourier Transform (FFT); Single-path Delay Feedback (SDF); Multiple-path Delay Commutator (MDC); Hybrid Architecture

Summary:

Pipelined FFT architectures are divided into series, parallel, and hybrid architectures. Series architecture offer small area at the expense of low throughput while parallel architecture offer high throughput at the expense of large area. Hybrid architectures are an intermediate point between series and parallel architecture where we can tune area and throughput requirement. Our work proposes a novel generic hybrid architecture for radix-r FFT. The proposed architecture offers great savings in memory requirements compared to currently available architectures using normal input order.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

Name: Mahmoud Ismail Soliman Nazmy Elbeltagy

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Dedication

To my wife for her continuous support and my daughter who is my strongest motivation to move forward and provide her a quality life.

Acknowledgments

I would like to thank my continuously supporting supervisor Dr. Omar Nasr who stood beside me, encouraged me, and pushed me towards submitting my thesis. Additionally, I would like to thank our god father Dr. Hossam Fahmy for his insights, guidance, and support and encouragements.

Finally, I would like to thank my family for continuously supporting me and pushing me forward. I would like also to thank my friends and colleagues, specially EECE department teaching assistants, for their help, guidance, and support.

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