



Cairo University

# **APPLICATION OF LIMIT ANALYSIS TO THE SLOPE STABILITY PROBLEM**

By

**Ahmed Mohameden Mahmoud**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
**CIVIL ENGINEERING - PUBLIC WORKS**

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
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Under the Supervision of

**Prof. Dr. Ashraf Kamal Hussein**



Professor of Geotechnical Engineering  
Department of public works  
Faculty of Engineering, Cairo University

**Dr. Sherif Adel Akl**



Associate Professor of Geotechnical Engineering  
Department of public works  
Faculty of Engineering, Cairo University

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Approved by the  
Examining Committee



**Prof. Dr. Ashraf Kamal Hussein**

Thesis Main Advisor



**Prof. Dr. Abdel-Salam Mohamed Salem**

Internal Examiner



**Prof. Dr. Amira Mohamed Abdel-Rahman**

External Examiner

Professor at Housing and Building National  
Research Center

**Engineer's Name:** Ahmed Mohameden Mahmoud  
**Date of Birth:** 20/6/1989  
**Nationality:** Egyptian  
**E-mail:** Mohameden\_ahmed@yahoo.com  
**Phone:** 01009575257  
**Address:** Giza, Egypt  
**Registration Date:** 01/03/2012  
**Awarding Date:** ..../..../2018  
**Degree:** Master of Science  
**Department:** Civil Engineering - Public Works



**Supervisors:**

Prof. Dr. Ashraf Kamal Hussein  
Dr. Sherif Adel Akl

**Examiners:**

Prof. Dr. Ashraf Kamal Hussein (Thesis main advisor)  
Prof. Dr. Abdel-Salam Mohamed Salem (Internal examiner)  
Prof. Dr. Amira Mohamed Abdel-Rahman (External examiner)  
(Professor at Housing and Building National Research Center)

**Title of Thesis:**

APPLICATION OF LIMIT ANALYSIS TO THE SLOPE STABILITY PROBLEM

**Key Words:**

Slope Stability; modeling; Seepage; Limit Analysis, Limit equilibrium

**Summary:**

This thesis aims to evaluating the stability of slopes using Limit analysis (LA) method and compare it to Finite Element Analysis (FEA) and conventional methods of analysis. Mohr-Coulomb soil model is used in the numerical modeling using assumed values of soil strength parameters to cover a wide range of different soil types. In this thesis, a series of numerical models are carried out using SLIDE software for Limit equilibrium (LE) analysis, slope stability charts based on LE methods for seepage analysis and Optum G2 with plane strain conditions for FEA and LA. Different cases of analysis are modeled regarding soil types, soil strength parameters and water head in the slope. The results are discussed and summarized regarding factor of safety.

  
Sherif Akl

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# **Dedication**

To the one who gracefully gave me birth and taught me everything, to my mother.

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## **Abstract**

Analysis of slopes has traditionally been carried out by limit equilibrium (LE) methods. These methods gain their popularity because they have proved to be reasonably reliable in assessing slope stability and they require few and attainable input parameters. LE methods assume that collapse will follow a particular pre-assumed geometry, which are effective for simple geotechnical problems, yet it may encounter difficulties when considering more complicated problems. Finite element analysis (FEA) is a more powerful method because it visualizes the stress distribution in the field. FEA is preferable in analysis of slopes with complex geometry, and variety of pore water pressure condition, external loads, and internal soil reinforcement. However, FEA in many cases is a cumbersome analysis and is affected by simplified assumptions and uncertainty of parameters.

Limit analysis (LA) can be an effective means of evaluating the stability of slopes without the limitation of an assumed slip surface and with less computational cost than FEA. In this research a comparison is given between the use of LE, FEA and LA for an idealized slope example. The results show that LA combined with optimization techniques can lead to results as accurate as FEA. Comparison also shows that LE computes higher factor of safety (FOS) than LA/FEA in undrained condition; and the results are almost the same in case of drained condition. When ground water flow is considered, LE methods are conservative compared to LA/FEA in a slope stability problem.

# Chapter 1 : Introduction

Evaluating the stability of slopes is an important as well as an interesting and challenging aspect in geotechnical engineering. Over many decades, experience with the behavior of slopes, and with their modes of failure, has led to development and improvement of our understanding to the changes in soil properties that can occur through the lifetime of slope, it is always important to determine the changes in the soil properties and the various loading and seepage conditions that may occur with time to decide the safety of a certain slope either it is a manmade or naturally found among the inhabitant area.

## 1.1. Background

Instability related issues in engineered slopes as well as natural slopes include serious challenges to both researchers and professionals. In construction areas, instability may result due to rainfall, increase in ground water table and change in stress conditions. Similarly, according to (Abramson L. W., Lee, Sharma, & Boyce, 2002) natural slopes that have been stable for many decades may suddenly fail due to various changes in geometry, external forces and loss of shear strength.

Although limit equilibrium analyses, as proposed, for example, (Janbu N. , 1954), (Bishop, 1955), (Morgenstern, N.R. and Price, V.E., 1965) and (Spencer, 1967), are widely used in practice of geotechnical engineering to calculate factors of safety, particularly in slope stability analysis. Alternatively, method such as the strength reduction technique (Zienkiewicz, Swan C.C, Y.K. Seo, 1975); (Griffiths, D. V. and P. A . Lane, 1999), which are traditionally based on the displacement finite element approach, have become increasingly popular. For slope stability problems, traditional methods as the limit equilibrium methods and finite element analysis are generally used to predict factors of safety (Cheng, Lansivaara, & Wei, 2007). However, based on the assumptions adopted in driving limit equilibrium analysis methods. These methods do not always result in a unique factor of safety and therefore they are unsuitable for generating a reference solution for assessing the accuracy of alternative methods.

In contrast, limit analysis LA can provide upper and lower bounds of the factor of safety (Sloan S. , 1988) and (Sloan, S.W., and Kleeman, P.W, 1995) where the true solution lies between two values and is therefore examined in this research and compared with those from limit equilibrium and the finite element strength reduction technique. The validity of this approach is investigated for a slope stability problem with a variety of conditions and soil types.

## 1.2. Scope and objective of the study

Many slope failures indicate that groundwater has an important effect on the slope stability. The coupling of fluid may lead is the internal cause of slope failure.

The importance of slope stability analysis is because the slope failure may interrupt some imperative services like traffic, water supply, power production. ect. In this way, the main incentive of stability analyses is to save human lives and reduce property damages.

In General, the basic cause of slopes instability is that the shear strength of the soil is less than the shear strength required for equilibrium. This condition is reached by either a decrease in the shear strength of the soil, or an increase in the soil shear stress required for to maintain equilibrium. (Duncan & Wright, 2005) stated that when a slope fails, it is usually not possible to decide the single reason that resulted in instability. For example, water influences the stability of slopes in many different ways that makes it impossible to isolate one effect of water and identify it as the cause of failure

Many methods are available for analyzing stability of slopes they include simple equations based on basic statics, charts, and slope stability computer programs based on finite element method. Generally, they are classified into the following three categories:

- Limit equilibrium approach
- Finite element method
- Limit analysis approach based on plasticity limit theorems

In this thesis Limit Equilibrium Methods, Finite Element and Limit Analysis Methods are used for the analysis. Optum G2 is a commercial geotechnical software program used for the finite element and limit analysis, SLIDE is used for limit equilibrium methods, Optum G2 is also used for Limit analysis in addition to slope stability charts which were developed based on classical limit equilibrium methods.

### **1.3. Organization of the thesis**

Presentation of this thesis has been organized in several Chapters. A brief description is given here. Chapter 2 describes the literature review on stability analysis methods. The review focus on the LE principles in FOS determination. Moreover, most of the LE methods are discussed with highlights on their fundamental differences and limitations in practical applications. The use of FE analysis and LA have also been discussed in Chapter 2. Comparisons of the LE, FE and LA have been given in Chapter 3, where a simplified and idealized slope has been analyzed.

Three conditions have been considered to compare the selective methods:

- Dry sand slope,
- Dry clay slope, and
- Wet sand slope with variable ground water level.

Finally, the respective FOS obtained from the LE, FEA and LA has been compared.

Chapter 4 includes discussions and comments on the applied methods, specific stability conditions due to various conditions and comparison of LE, FE and LA methods. The discussion is also related back to the highlights from the comparisons carried out in Chapter 3.

Finally, the main findings on the slope stability evaluations from the study are summarized in Chapter 5, followed by recommendations for further research.

# Chapter 2 : Literature Review

## 2.1. Introduction

The problem of slope stability is a typical problem in the geotechnical engineering, this is because the foundation works are usually carried out around sloping soil profiles and also because the need of excavating or filling creates slopes that can be future hazard. The soil slopes gain their stability from the existence of shear strength enough to balance the existing loads on the soil itself (i.e. gravity and surcharge load) and the seepage force of moving water. Limit equilibrium methods are typically used along with other methods in most common types of slopes.

The choice of the method of calculation whether to use effective or total stress analysis and which values of shear strength to use in the analysis, and based on the analysis type an estimated value is calculated for factor of safety.

## 2.2. Limit equilibrium methods

All limit equilibrium methods based on their derivation on Mohr-Coulomb material model. (Janbu, 1973) defined the factor of safety of a certain slope by the ratio between the available shear strength to the mobilized shear stress in an equilibrium condition. According to (Duncan & Wright, 2005) two different approaches are used to satisfy static equilibrium in the limit equilibrium analysis.

- Single free body diagram
- Method of slices

Single free body diagram consider equilibrium for the entire rigid mass of soil bounded between an assumed slip surface and the top surface of the slope. In this method, equilibrium equations are solved for a single rigid free body. Examples of this method are the Swedish slip circle method and Log-spiral procedure. Such methods may not be appropriate for complicated slope geometry and if the slope involves structural elements such as slope stabilizing piles soil nailing or geogrids.

Method of slices: In other procedures the soil mass is divided into an integer of vertical slices and the equilibrium equations are written and solved for each slice individually. These procedures, termed procedures of slices, examples for such methods are the Ordinary Method of Slices, the Simplified Bishop procedure, and Spencer's Procedure and other methods are discussed in the below section. Some procedures use and satisfy all of the three equilibrium equations (equilibrium of forces on the horizontal and vertical direction, and equilibrium of moment at any point), others use and satisfy only some.

Whether the equilibrium equations are solved for a single rigid body or for an individual vertical slices combining together to create the entire sliding block, the number of unknowns is more than the number of equilibrium equations; hence, the problem is statically indeterminate. Therefore, in order to achieve a balanced number of equations and the equilibrium equations assumptions shall be made. Different methods based on