



# **BOUNDARY ELEMENT FORMULATIONS FOR NONLINEAR SOIL-STRUCTURE INTERACTION**

By

**Ahmed Fady Mahmoud Farid**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**DOCTOR OF PHILOSOPHY**  
in  
**Structural Engineering**

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FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
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**Title of Thesis:**

Boundary element formulations for nonlinear soil-structure interaction

**Key Words:**

Boundary element method; Soil-structure interaction; Nonlinear analysis; piled raft;  
Overall building analysis

**Summary:**

In this thesis, a comprehensive study to overcome soil-structure interaction problems that are present in nonlinear analysis of piled rafts. Direct boundary element formulations are used to perform nonlinear analysis of piled rafts. The raft is considered as a shear-deformable plate, whereas the soil is modeled as either Winkler springs or elastic half space (EHS). Piles are modeled using their actual geometry including all interaction effects. Two different direct boundary element formulations are reviewed: a) Plate stiffness matrix formulation (FEM-like formulation), and b) adding stiffness matrices to the plate integral equation formulation (BEM-like formulation) are used.

Firstly, the proposed BEM plate stiffness matrix formulation is extended to analyze plates on elastic-plastic Winkler foundations. Iterative methodology is used to perform nonlinear analysis of plate on elastic-plastic Winkler foundations. This methodology is also extended to analyze plates on elastic-plastic and tensionless Winkler foundations.

Subsequently, nonlinear analysis of piled rafts is developed. Nonlinear analysis of piled rafts at certain load level is developed using three different formulations and two nonlinear numerical techniques. These formulations are automated and implemented in *PLPAK* software under title of "full load analysis". In addition, two nonlinear formulations are extended to predict the load-settlement curve of the piled raft. The load-settlement curve prediction is implemented in the *PLPAK* software under title of "failure incremental analysis". Interface is developed to help user in the analysis called "*NLPAK*" (Nonlinear package).

Finally, overall building analysis system using *PLPAK* software is presented. Innovative coupling technique of the super and substructure is presented to analyze overall building. Coupling technique is implemented into *PLPAK* software. Nonlinear package developed in thesis is coupled with superstructure package to analyze overall building. Laterally loaded piles are considered in approximate way. In addition, coupling technique is used to study stage of construction analysis including soil structure interaction on the overall building, which yielded interesting results.

Several numerical examples and practical examples are presented to verify the proposed techniques. Results are compared to commercial software packages and previously published results. The examples provide proof of concept, yet push the limits of accurate numerical analysis of overall structures on nonlinear soil.

## Acknowledgments

I would like to express my very great appreciation to Professor **Youssef F. Rashed** for his valuable and constructive suggestions during the planning and development of this research work. His willingness to give his time so generously has been very much appreciated. He is not only a supervisor; but he is also give me advices in my life like my father.

I would like also to thank **my mother** without whom I was not be able to be a researcher in faculty of engineering, Cairo University.

I also wish to express my deepest gratitude to **my lovely wife Asmaa Magdy** who are always behind me for the success. I would not have achieved this work without her help and participation.

I should also thank my dear friend **Ahmad Torky** for his great help in this work in many parts, in addition to supporting me during the research project of this work.

I also thank **Ahmed Mahmoud (Taiko)** to his help in this work. Also, all **CUFE-BE members**, who work as one great research team, especially **Mahmoud El Galad** and **Osama Eid**.

I also wish to thank **my father**, and **my dear brothers Amgad and Eslam** who are always encourage me to success.

I also should thank **Magdy Aly** for his great help in covering me in many critical situations.

I also should thank my lovely son **Youssef Ahmed Fady** to can stand away from me for a long time.

Finally, This thesis is an outcome of a project that supported financially by the Science and Technology Development Fund (**STDF**), Egypt, grant no 14910 . The author would like to acknowledge the support of (**STDF**).

## *Dedication*

*I am dedicating this thesis to three beloved people who have meant and continue to mean so much to me.*

*First and foremost, to my mother **Hoda** although she is no longer of this world, their memories continue to regulate my life, whose love for me knew no bounds and, who taught me the value of hard work, Thank you so much “Mama”, I will never forget you, and I hope you to be with me in these moments.*

*Next, my wife “**Semsem**” **Asmaa** who raised me, loved me, and support me. Whish Allah give us a happy life with you forever.*

*Last but not least I am dedicating this to my baby **youssef ahmed fady** to encourage him to be a man bears responsibility.*

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