hossam maghraby

شبكة المعلومات الجامعية

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التوثيق الالكتروني والميكروفيلم

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صفحات بالرسالة

لم ترد بالأصل

ON TESTING CLASSES OF LIFE DISTRIBUTIONS AGAINST VARIOUS ALTERNATIVES

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By

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from the Institute of Statistical Studies and Research,

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CONTENTS

I. INTRODUCTION

Ageing is a phenomenon that has implications on many research areas such as life testing, reliability studies, actuarial science, economics, manufacturing, engineering, medicine and pharmaceutical studies among many. At the macro level, states or economic systems like to categorize "old age" since this is an economic dependent state that puts responsibilities on governments and organizations. While at the micro level, a manufacturer would like to know how long a warranty of a product should run, and when to end before the inception of "old age" of the product when lots of maintenance and or repairs become necessary. In medicine, survival analysis and life testing has always centered on the concept of ageing mostly through identifying classes of ageing phenomena. Defining classes that characterize positive ageing has long been an area of active research. Early developments are embodied in the book of Barlow and Proschan (1981) where early ageing concepts and classes such as "increasing failure rate," (IFR), "increasing failure rate average" (IFRA), "new is better than used" (NBU), "new is better than used in expectation" (NBUE), and the "decreasing mean residual lifetime" (DMRL) are defined and studied.

During the past several decades various classes of life distributions have been pro-posed in order to model different aspects of aging . The best known of theses classes are IFR, IFRA, DMRL, NBU, NBUE and HNBUE. The relations between these classes are the following .

$$\text{IFR} \subset \text{IFRA} \subset NBU \subset NBUE \subset HNBUE;$$

$$\text{IFR} \subset DMRL \subset NBUE \subset HNBUE.$$

Definitions, properties and applications of these aging notions can be found, for instance, in Bryson and Siddiqui (1969), Barlow and Prochan (1981), Klefsjo (1982), Ahmed (1990) and Alzaid (1994).

In Chapter II, we present and classify most of the aging notions and families of life distributions that are based on these notions.

Sec. 2.2.1 considers classes defined through failure rate functions. Classes defined through the conditional survival probabilities are given in Sec. 2.2.2. Sec. 2.2.3 presents families based on conditional mean residual lives. Families based on connections between parent distributions and their equilibrium distributions are given in Sec. 2.2.4. Sec. 2.2.5 considers classes of life distributions associated with specific partial orderings of interest in statistics. Sec. 2.2.6 gives some discrete classes of life distributions. Sec. 2.2.7 gives classes based on the concept of long tail (asymptotic decay).

Preservation properties under commonly occurring operations in reliability are presented in Sec. 2.3. Classical testing of being a member in one of the presented families of life distributions is given in Sec. 2.4.

In **Chapter III,** new classes of life distributions are introduced and their properties are studied. In particular the families: new better than aged (NBA), new better than aged in expectation (NBAE) and harmonic new better than aged in expectation (HNBAE) are defined and their closure properties under convolutions, mixtures, shock models and the characterization under Laplace transform are established. Each of the above concepts is based on the notion of long tailed distributions (asymptotic decay).

The NBA class is considered in Sec. 3.1. Its preservation properties are established in Sec 3.2, Sec. 3.3 and Sec 3.4. Discrete analogs of introduced distributions are defined and their closure properties under shock models are studied in the same section. Characterization of some of these classes in term of Laplace transforms are developed in Sec. 3.5.

In **Chapter IV**, test statistics (based on U-statistic) are built to test the hypotheses that F belongs to the UBA family versus being exponential. The statistical properties of the test statistic are established. Its variance is calculated in sec 4.2 and its limiting distribution is derived in Sec. 4.3. In particular, a standardized form of the statistic is shown to have the asymptotic normal distribution (see Th.1 Sec. 4.3). Monte Carlo studies are calculated to determine the critical values of the suggested test statistic and its power are considered in Sec. 4.4 for samples of size $n = 5(1)25(5)50$. The Pitman relative efficiency of the test statistic is studied in Sec. 4.6.

Chapter V, reconsiders the testing problem but this time using the kernel methods rather than U-statistic. Details of the test procedure are presented in Sec. 5.2. Asymptotic properties of the test statistic have been also proved in the same section. Monte Carlo studies, similar to these of Sec. 4.4, for the kernel statistic are calculated and their results presented in Sec. 5.3. The power estimates of the test against several alternatives are examined and their results reported in Sec. 5.4. The Pitman efficiency is studied in sec. 5.5 and the asymptotic relative efficiency is presented (for the kernel method) in Sec. 5.6.

Finally, **Chapter VI**, points out several open points for future research.

II. CLASSES OF LIFE DISTRIBUTION FOR AGEING

Ageing is a phenomenon that has implications on many research areas such as life testing, reliability studies, actuarial science, economics, manufacturing, engineering, medicine and pharmaceutical studies among many. At the macro level, states or economic systems like to categorize "old age" since this is an economic dependent state that puts responsibilities on governments and organizations. While at the micro level, a manufacturer would like to know how long a warranty of a product should run, and when to end before the inception of "old age" of the product when lots of maintenance and or repairs become necessary. In medicine, survival analysis and life testing has always centered on the concept of ageing mostly through identifying classes of ageing phenomena.

We plan to address the two most interesting concepts in this area and those are studying ageing via defining classes of "positive ageing" and also defining and studying "old age." The two wings are interlinked and feed each other as we shall detail a bit later. Defining classes that characterize positive ageing has long been an area of active research.

Early developments are embodied in the book of Barlow and Proschan (1981) where early ageing concepts and classes such as "increasing failure rate," (IFR), "increasing failure rate average" (IFRA), "new is better than used" (NBU), "new is better than used in expectation" (NBUE), and the "decreasing mean residual lifetime" (DMRL) are defined and studied. Later on, the above classes of ageing were shown to emerge from the comparisons in several senses of three notions in ageing; namely, "life itself" (denoted by X), as well as the "random remaining life," which is the remaining life at age t (denoted by X_t), and the third is the "equilibrium lifetime" (denoted by X_e) which will be used as our definition of old age, for details see Shaked and Shanthikumar (1994). For the past three decades, life or ageing classes were introduced and studied via comparing X with either X_t or X_e. These comparisons took basically the forms of "dominance" of a certain order that for long have been known in economic studies as well as computational mathematics. Thus to define an ageing class, we say that X dominates X_t (or X_e) in some sense.

Most of the studies defining and or studying classes of ageing by comparing any two of X, X_t or X_e centered on either developing the probabilistic characters of the class or doing the statistical analysis of the class. Newly, however, the two avenues merged creating a common ground to work on and make a productive results.

2-2 Families of life distributions

During the past several decades various classes of life distributions have been pro-posed in order to model different aspects of aging . The best known of theses classes are IFR, IFRA, DMRL, NBU, NBUE and HNBUE. The relations between these classes are the following .

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