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Studies in $f(T)$ Theories

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by

Shymaa Khaled Ibraheem Abd El Ghany

Department of Mathematics, Faculty of Science
Ain Shams University

Supervised by

Prof. Dr. Mamdouh Ishaac Wanas

Department of Astronomy
Faculty of Science
Cairo University

Prof. Dr. Gamal Gergess Nashed

Faculty of Engineering
The British University in Egypt

Dr. Tarek Nasr El-dein Salama

Department of Mathematics
Faculty of Science
Ain Shams University

Department of Mathematics - Faculty of Science
Ain Shams University

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List of Abbreviations

ΛCDM	Λ-Cold Dark Matter - Standard Model of Big-Bang Cosmology
AP	Absolute Parallelism
B-B	Big-Bang
BB	Building Blocks
CMBR	Cosmic Microwave Background Radiation
COBE	Cosmic Microwave Background Explorer
DEC	Dominant Energy Condition
EoS	Equation of State
FRW	Freidmann-Robertson-Walker
GFT	The Generalized Field Theory
GR	The General Theory of Relativity
MD	Matter Domination
NEC	Null-Energy Condition
RD	Radiation Domination
SEC	Strong Energy Condition
TEGR	The Teleparallel Equivalent to General Relativity

WEC	Weak Energy Condition
WMAP	Wilkinson Microwave Anisotropy Probe

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Abstract

Despite many successes have been achieved by the Standard Friedmann-Robertson-Walker Cosmology, there are some other problems which are not solved, so far. We construct a bounce inflation model in a viable $f(T)$ modified gravitational theory. In this model, the Universe gracefully exit into the Standard Friedmann-Robertson-Walker decelerated Universe. We make use of the phase space technique to analyze the evolution of the Universe. We study the cosmic thermal evolution and show that hypothesized model predicts a supercold Universe during the pre-contraction phase. This result is consistent with the requirements of the slow-roll scenarios.

Moreover, we show that the proposed model performs a reheating period by the end of the contraction with a maximum temperature just below the grand unified theory temperature. However, it matches the radiation temperature of the hot big-bang at later stages. We show that the equation-of-state due to the effective gravitational sector suggests that constructed model is self-accelerated by teleparallel gravity. After that, we assume that the matter component is a canonical scalar field and study the slow-roll parameters. We obtain the scalar field potential induced by $f(T)$ gravitational theory. When we study the power spectrum of the model, we find that it is nearly scale invariant. Also, we show that the model under consideration unifies inflaton and quintessence fields in a single model. Finally, we revisit the primordial fluctuations in $f(T)$ bounce cosmology and study the fluctuations that are produced at the pre-contraction phase.

Summary

The Standard FRW (Big-Bang) Cosmology has succeeded to trace the cosmic thermal evolution in an elegant way by comparing the particles interactions rate with the expansion rate of the Universe. At very hot stages, the rate of particle interactions is much larger than the expansion rate of the Universe and local thermal equilibrium could be achieved. At later stages, when the Universe cools down, the interaction rate decreases faster than the expansion allowing the particles to decouple from the thermal path at the equality of the rates. On the other hand, the Standard big-bang Cosmology suffers many problems, e.g., *Initial Singularity*, *flatness*, *particle horizons*, etc. Solving these problems requires a superfast accelerated expansion phase at some early time, i.e., cosmic inflation [6, 57, 93, 116, 123], which is usually represented by an exponential expansion at $\sim 10^{-35}$ s after the big-bang. As a result, the Universe becomes isotropic, homogeneous and approximately flat. Standard inflation models assume the existence of a self-coupled scalar field (inflaton) minimally coupled to gravity, whose potential governs the evolution of the Universe during inflation. During this stage, the initial quantum fluctuations cross the horizons and transform into classical fluctuations producing a nearly scale-invariant scalar perturbations spectrum. Although inflation solves the above mentioned problems, one of the fundamental problems still exists, that is the *initial singularity* which arises when tracing the Universe back in time as divergences of the cosmic temperature and density. Since the initial singularity is before inflation raids, the problem can not be solved within inflationary Cosmology. Another serious problem is the *trans-Planckian* problem which also appears in inflationary cosmology where the cosmological scales that we observe at present time correspond to length scales smaller than the Planck length at the onset of inflation [22, 98].

One of the suggested alternatives is by assuming that the scale factor initially shrinks down to a nonzero minimal value then *bounce* to an expanding phase. In this case a singular or nonsingular bounce Universe can be obtained [29, 108]. This idea has been extended to recognize nonsingular cyclic Universe models, e.g., pre-big-bang [55]. Other than the nonsingular issue, bounce cosmologies have many interesting features such as solving the horizon and flatness problems even in the initial shrinking phase. Also, these models can generate scale-invariant scalar perturbations as supported by observations. However, bounce models are usually faced by two main problems [7, 127]: The first is called the *anisotropy* problem, that is in the contraction phase the anisotropies grow faster than the background, so that the contraction ends with a complete anisotropic Universe which violates the cosmological principle and bouncing to an expanding phase will not occur. The second is called the *ghost instability* problem, that is the bounce cosmology violates the null energy condition (NEC), which gives rise to ghost degrees-of-freedom. However, both two issues have been successfully resolved within a nonsingular bounce cosmology [26, 30, 31].

The above mentioned anisotropy problem can be deluded if the equation-of-state parameter is larger than unity during contraction, then the background dominates the anisotropies. Indeed, a large equation-of-state parameter constrains the potential to be *negative* in scalar field models. On the other hand, the ghost degrees-of-freedom is an outcome of using the GR theory, while other modified gravity theories could alter the situation (for reviews on modified gravity theories, see, for instance, [12, 14, 34, 41, 48, 78, 85, 106, 107]). In $f(T)$ modified gravity theories, where T is the torsion scalar described by the Weitzenböck connection in the teleparallelism [11, 52, 54, 70, 71], it has been shown that nonsingular bounce solutions can be constructed in a straightforward way [28, 29, 32]. Also, it has been shown that $f(T)$ gravity combined with holonomy corrected loop quantum cosmology supports the bounce Universe model [7, 58–60].

Constructing a viable bounce $f(T)$ model is the main object of this thesis and it is discussed in details in Chapter 3, where we propose a possible choice of a scale factor that is capable to perform a reliable cosmological model with two possible scenarios: a graceful exit inflation or a bounce graceful exit inflation. Chapter 3 is dedicated then for corresponding evolution and phenomenology where we use the phase space to study the thermal evolution of the Universe.

The thesis has the following structure:

Cahpter 1: The Standard FRW Cosmology

Einstein General Theory of Relativity (GR) is presented in some details. We start with its motivations and its two main covariance and equivalence principles. Then, its main features and formulation in Curved Geometry of Riemannian Space are given. The Action is written and Field Equations are then derived using the least action principle. Many successes of GR are then exhibited, namely, applications into the Solar System dynamics including precession of planets and other GR tests and the successes in Cosmology for the expansion of the Universe. On the other hand, some problems of GR in its cosmological applications were discussed. Specifically, we discuss accelerating expansion of the Universe, the particle horizons and the flatness problems of the world models. Then, we revise the idea of inflation as a potential solution for the horizons and flatness problems. We discuss the slow roll case in details and deduce the slow-roll conditions and define the observable quantities from inflation. Finally, we discuss the quadratic potential for inflation and the reheating mechanism after inflation.

Cahpter 2: Modified Gravity Theories

The $f(R)$ and $f(T)$ modified gravity theories are reviewed. First, the $f(R)$ action and field equations are introduced and the equivalence of $f(R)$ with Brans-Dicke theory is discussed. After that, some elements of the cosmological phenomenology of $f(R)$ are introduced and an effective equation-of-state parameter is deduced. For illustration, the power R^n example is considered. Then the $f(T)$ modified gravity is introduced. We review first the teleparallel gravity equivalent theory of GR (TEGR) by reviewing the basic elements of the AP-Space in four dimensions by introducing its basic structure components, the tetrads. We then define the torsion, contortion and superpotential tensors and the torsion scalar in terms of them. After that, the action of $f(T)$ is introduced as a direct generalization of the TEGR action. Finally, the $f(T)$ field equations are derived in details.

Cahpter 3: A Suggested Bounce Inflation Model in $f(T)$ Cosmology

The work is organized as follows. In Section 3.1, the $\dot{H} - H$ phase space of the FRW Cosmology is discussed in some details. In Section 3.2, we discuss a possible choice of a scale factor capable to perform a reliable cosmological model. We show that two possible scenarios could be used according to the values of the model parameter: a graceful exit inflation or a bounce graceful exit inflation. Also, we use the nice feature of $f(T)$ cosmology to represent the modified Friedmann equation as a one-dimensional autonomous differential equation. This enables to construct the corresponding $\dot{H} - H$ phase space, where the dynamical evolution of the model can be exhibited clearly. In Section 3.3, we construct an $f(T)$ theory corresponding to the bounce inflation model. Also, we evaluate the equation-of-state of torsion gravity showing its role to describe a healthy bounce Universe. In Section 3.4, we discuss the thermal evolution of the Universe showing that its maximum reheating temperature is at the bounce point. We show how the slow-roll condition can arise naturally in this model as a consequence of its thermal evolution. We assume that the matter component of the Universe is a canonical scalar field, and then we obtain the potential corresponding to the proposed $f(T)$ theory. The slow-roll potential provides a nearly scale invariant spectrum consistent with observations. So the proposed model does not suffer from a large tensor-to-scalar ratio that is usually obtained in bounce scenarios. In addition, we show that for a particular case, the model can unify inflaton-quintessence fields in a single model. We also show that the null-energy condition is not generally violated, which makes the model safe from the ghost instability problem. In Section 3.5, we extend our analysis to investigate the $f(T)$ theory at the perturbation level to study the primordial fluctuations during the precontraction phase. The work has been summarized and concluded in Section 3.6.

Also, a list of references is included.

The main results of the thesis are published in the joint paper

K. Bamba, G.G.L. Nashed, W. El Hanafy, Sh.K. Ibraheem, *"Bounce inflation in $f(T)$ Cosmology: A unified inflaton-quintessence field"*, Phys.Rev. D94 (2016) no.8, 083513 (arXiv:1604.07604 [gr-qc])