Ain Shams University

Girls College for Arts, Science and Education

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ON RECENT METHODS FOR SOLVING MULTICRITERIA DECISION MAKING PROBLEMS

A Thesis

Submitted to Girls College for Arts, Science and Education,

Ain Shams University, Mathematics Department

For the Degree of

Ph.D.of philosophy

(Mathematics)

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ACKNOWLEDGEMENT

Praise be to Allah for giving me the courage and fortitude to complete this work. I would like to express my profound gratitude to prof.Dr.Mohamed Osman for suggesting fruitful points of research guidance ,devotion and promotion for completion of this work .AlsoI would like to express my happiness to be a student of him as a great scientist in this branch of mathematics. I would like to express my profound thanks toprof.Dr.Samiya El Azab for her greet efforts ,her time and her kind help during our work. AlsoIwould like to thank .to .Dr.MonamohamedM.M.Tarrad for her greet efforts, her time and her kind help during our work. Also I would like to thank him for giving me a lot of important scientific advises during this thesis. Thanks to all members of mathematic department girls college for arts science and education.

CONTENTS

Chapter 3 **FUZZY BASICNOTIONS IN FUZZY PARAMETRIC MULTIOBJECTIVE NONLINEAR PROGRAMMING PROBLEMS.**

Chapter 4 INTERACTIVE APPROACH FOR OPTIMAL SELECTION PROBLEM.

SUMMARY

Multiple criteria decision making problem (M-CDMP) is concerned with methods and procedures by which multiple criteria can be formally incorporated into the analytical process.

The (M-CDMP) arises in wide variety of problems, such as vector maximization , goal programming , group decision problems (with several criteria), multi- attribute problems, and utility and theory of measurements. Vector optimization problems (VOP) or multiobjective optimization problems are one main branch of mathematical optimization. (VOP) appear when a decision maker must take a decision satisfying the optimization of more than one conflicting objectives.

Stability analysis in multiobjective nonlinear programming has been extensively investigated from the qualitative point of view . The stability notions are the sets of parameters that retain specific features for the optimal solutions of the multiobjective nonlinear programming problems.

This thesis consists of five chapters:

Chapter 1 presents a survey on Multiplecriteria decision making problems. Some different methods for dealing the problem, such as the goal programming approach , vector optimization problem and interactive approach are introduced Some definitions of fuzzy theory are presented.

Chapter 2 the concept of dual parametric problems is discussed to clarify the fruitful relation between the primal and dual problems. The discussion of the possibility solving one of them , where the second problem clearly solved,where parameters are in the objectives,are in the constrain,or in both .

Chapter 3 is devoted to Fuzzy parametric multiobjective nonlinear programming problem with fuzzy parameters in the objective functions, with fuzzy parameters in the constraints , and in both the objective functions and the constraints .

Chapter 4 presents an interactive approach for the three optimal selection problems where three decision makers (DM_s) are involved in the selection of single offer from a pool of n offers.

Chapter 5 is devoted to the conclusions arrived in carrying out this thesis and some suggestions for further research are given.

CHAPTER 1

SURVEY ON THE RELATED TOPICS

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1.1 Introduction

There had been a growing interest and activity in the area of multiple criteria decision making (MCDM) , especially in the last 20 –years . Modeling and optimization methods have been developed in crisp and in both fuzzy and Rough environment . Many problems in MCDM are formulated as multipleobjective mathematical programming problems . Some examples of these such as natural resource management problems , project design problems and financial planning problems .

In the crisp environment , there are many approaches for solving the MCDM problems , form these approaches utility theory , vector optimization , goal programming and interactive approaches .

These types of approaches have been discussed by Chankong and Haims [1] , Fishburn [2] ,Ignizio[3] Sawaragi, Nakayama and Tanino[4] , Shin and Ravindran [5,6] and Steuer [7].

In a fuzzy environment , Tanaka and Asai [8] ,formulated multiobjective linear programming problems with fuzzy parameters and Orlovski [9] formulated general multiobjective non linear programming problems with fuzzy parameters. Recently, Sakawa and Yano [10,11] introduced the concept of α-multi-objective nonlinear programming and α -pareto optimality . The concept of α -pareto optimality is introduced by extending the ordinary pareto optimality on the basis of the α -level sets of the fuzzy numbers .From this concept (for a certain degree α) ,the fuzzy multi objective programming problem can be understood as the crisp multiobjective programming problem .This chapter consists of three sections (section1.2 and section1.3,1.4)

Section 1.2 introduced the basic theoretical concepts and methods for treating MCDM in the crisp environment. In section 1.3 the definitions and terminology of fuzzy subsets are presented . In section 1.4 the definitions and terminology of rough subsets are mentiond. .

1.2 Multi criteria Decision Making Problems

Decision making is an integral part of our daily life . It considers situations ranging in complexity form the simple to the most complex involving multiple objectives .

1.2.1 Definitions

Multiobjective programming (MOP) problems or vector optimization problems (VOP) arise when there are two or more objectives to be optimized simultaneously . A mathematical formulation of a VOP is

$$\text{(VOP)} \quad \text{Min} \quad F(x) = (f_1(x), f_2(X), \ldots\ldots. f_k\ x))$$

$$\text{Subject to} \quad x \in X ,$$

Where

x is an n - dimensional vector of decision variables ,

X is the decision space or the feasible region ,

$X = \{ x \in R^n \ / \ g_s(x) \leq 0 , s = 1,2,\ldots..,m \}$,

$g_s(x)$, $s = 1,2,\ldots..,m$, are the constraints ,

$F(x)$ is a vector of k real valued functions defined on X ,

"Min " requires that $F(x)$ be minimized with respect to the preferences of the decision maker (D M) .

In order to make the problem nontrivial ,it is assumed that the objectives are in conflict and incommensurable . Due to the Conflicting nature of the objectives , an optimal solution that simultaneously minimizes all the objectives is usually not obtainable . Instead there are several solutions , called efficient solutions (non inferior solution or pareto-optimal solutions). Wich can be defind as follows

Definition 1.1

A point $x^* \in X$ is said to be an efficient solution of VOP if there is no $x \in X$ such that $f_i(x) \leq f_i(x^*)$ $i = 1,2,....,$ k with at least one strict inequality .

Definition 1.2

A point $x^* \in X$ is said to be a weak - efficient solution of VOP if there is no $x \in X$ such that $f_i(x) < f_i(x^*)$ $i = 1,2,....,k$.

Clearly , if x^* is an efficient solution of VOP then x^* is a weak - efficient solution of VOP .

Definition 1.3

A point x\` is said to be a properly efficient solution of VOP iff x \` is an efficient solution of VOP and there exists a scalar $M > 0$ such that for each i and $x \in X$ satisfying $f_i(x) < f_i(x`)$ , we have $(f_i(x`) - f_i(x)) / (f_j(x) - f_j(x`)) \leq M$ ; for some i such that $f_i(x`) < f_i(x)$.

Definition 1.4

The ideal solution for VOP is obtained by minimizing each objective function separately under the given constraint .

For the (V O P) problem , the ideal solution is the vector

$f^* = (f_1^* ,f_2^*,......, f_k^*)$, wher $F_i^* = \min_{x \in X} f_i(x)$, $i = 1,2,.....,k$,

Definition1.5

A function U , which associates a real number U(F(X)) to each x $\in$ X , is said to be a utility function representing a particular decision makers preference structure provided that :

1. $F(x^1) \sim F(x^2)$ iff $U(F(x^1)) = U(F(x^2))$ for $x^1, x^2 \in X$; and

2. $F(x^1) \succ F(x^2)$ iff $U(F(x^1)) > U(F(x^2))$ for $x^1, x^2 \in X$;

Where $F(x^1) \sim F(x^2)$ denotes that the decision makers is indifferent between outcomes $F(x^1)$ and $F(x^2)$,and $F(x^1) \succ F(x^2)$ denotes that the decision maker prefers $F(x^1)$ to $F(x^2)$.

Remark . A utility function is also known as a value function or preference function .

Definition 1.6

The local tradeoff ratio (indifference tradeoff or marginal rate of substitution) between f_i and f_j at point x^* is

$$\frac{\partial u / \partial f_j(x)}{\partial u / \partial f_i(x)} \quad , \; x = x^*$$

Finding the solution to the VOP reduced to the problem of finding all efficient solutions .Often there are an infinite number of efficient solutions and they are not comparable, hence , it is assumed that the decision maker has a utility function U , and with this assumption the VOP is reduced to :

Max $U(f_1(x), f_2(X), \ldots\ldots f_k(x))$

Subject to $x \in X$.

Note that if a utility function could be easily found for each multi objective mathematical program , there would be no need for multiobjective optimization techniques . Every VOP could be restructured as a single objective problem .

Decision makers with different preferences have different solutions for a vector optimization problem based on the preferences . This solution is commonly referred to as the best programming methods attempt to find a best compromise solution by progressive articulation of preferences solution that is also efficient solution, while other methods lead to a best compromise solution which may not be efficient solution ,([1]).

1.2.2 Common approaches to characterizing efficient solutions

The most common strategy to characterize efficient solutions of VOP in terms of optimal solutions is scalarizing the problem . Among the many possible ways of obtaining a scalar problem from a VOP , the following :

i. **The weighting problem**

Let $W = \{ w \in R^k \ / \ w_j \geq o$ and $\sum_{j=1}^{k} w_j = 1 \}$ be the set of nonnegative weights . The weighting problem is defined for some $w \in W$ as $P(w)$,

$$(P(w)) \qquad \text{Min} \ \sum_{j=1}^{k} w_j f_j(x).$$
$$x \in X$$

Theorem 1.1

Assume X is a convex set and f_j, $j = 1,2....,k$ are convex functions defined on X . If x^* is an efficient solution of VOP , then there exists $w \in W$ such that x^* solves $P(w)$.

Theorem 1.2

x^* is an efficient solution of VOP if there exists $w \in W$ such that x^* solves $P(w)$ and if either one of the following two conditions holds :

1. $w_j > 0 \ \forall \ j = 1,2 \ ...., k$, or

2. x^* is the unique solution of $P(w)$.

ii. **The r -th objective Lagrangian problem**

The efficient solutions of VOP can be characterized in terms of the optimal solution of the following form :

($P_r(l)$) Min $f_r(x) + \sum_{\substack{j=1, \\ j \neq r}}^{k} l_j f_j(x)$,

$x \in X$

where $l \in L_r = \{(l_1, l_2, l_3, \ldots, l_{r-1}, l_{r+1}, \ldots, l_k) \,/\, l_j \geq 0$ for each $j \neq r\}$.

Theorem 1.3

x^* is an efficient solution of VOP if for some r there exists $l \in L_r$ such that x^* solves $p_r(l)$, and if either

1. $l_j > 0$ for all $j \neq r$, or

2. x^* is unique minimize of $P_r(l)$.

iii. **The r-th objective ϵ -constraint problem**

The r-th objective constraint problem is formulated by taking the r - th objective function f_r as the objective function and letting all the other objective functions f_j ($j \neq r$) be inequality constraints . That is , the r -th objective **ϵ** -constraint problem can be defined as the scalar optimization problem $P_r(\epsilon)$,

$P_r(\epsilon)$ Min $f_r(x)$

Subject to $f_j(x) \leq \epsilon_j$, $j = 1,2,\ldots,k$, $j \neq r$,

where $\epsilon = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{r-1}, \epsilon_{r+1}, \ldots, \epsilon_k\}$. For a given point x^*, we shall use the symbol $p_r(\epsilon^*)$ to represent the problem $p_r(\epsilon)$, where $\epsilon_j = \epsilon_j^* = f_j(x)$, $j \neq r$.

Definition 1.7

The sensitivity function $\phi_r(y)$ of $p_r(\epsilon)$, where

$\phi_r : R^{K-1} \to \bar{R} = [-\infty , +\infty]$ is defined as

$$\phi_r(y) = \inf_{x \in X} \{ f_r(x) / f_j(x) - \epsilon_j \le y_j , j \ne r \}.$$

Definition 1.8

$p_r(\epsilon)$ is said to be stable if $\phi_r(0)$ is finite and there exists $M > 0$ such that for all $y \ne 0$,

$$\frac{\phi r(0) - \phi r(y)}{\| y \|} < M .$$

Theorem 1.4

A point x^* is an efficient solution of VOP iff x^* solves $p_r(\epsilon^*)$ for every $r = 1, 2, \ldots, k$.

Theorem 1.5

If x^* is a unique solution of $p_r(\epsilon^*)$ for some $r \in \{1, .., k\}$, then x^* is an efficient solution of VOP .

Theorem 1.6

Assume that $f_1(x), \ldots\ldots, f_k(x)$, are convex functions on the nonempty convex set X and that x^* is an efficient solution of VOP . Then x^* is properly efficient solution of VOP iff $p_r(\epsilon^*)$ is stable for each $r = 1, 2, \ldots, k$.

Definition 1.9

The functions $g_s(x)$, $s = 1, 2, \ldots, m$,which defines the convex feasible region $X = \{ x \in R^n / g_s(x) \le 0, s = 1, .., m\}$ are said to satisfy :

a. Slater's constraint qualification (on R^n) if there exists $x` \in R^n$

such that $g_s(x`) < 0$ for all $s = 1, 2, \ldots, m$.

b. Karlin's constraint qualification (on R^n) if there exists no

$p \in R^n$, $p_s \ge 0$ for all $s = 1, 2, \ldots, m$ with strict inequality for at least one s , such that $\sum_{s=1}^{m} p_s g_s(x) \ge 0$.