



# USING INTERVAL ARITHMETIC FOR ELECTRONIC CIRCUITS SIMULATION

By

Amin Maher Abdallah Baraka

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Electronics and Communications Engineering

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**Title of Thesis:** **Using Interval Arithmetic for Electronic Circuits Simulation**

**Key Words:** interval arithmetic, compact device models, circuit simulation, Monte-Carlo, design variability

### **Summary:**

Semiconductor devices scaling provides higher circuit density and faster devices, this provides better performance and more functionality for electronic chips. Variations affect circuit behavior as more as device scaled down. Verification of circuit behavior under the uncertainty arises from different variations is a challenge. Monte-Carlo statistical analysis and corner case analysis are used to estimate the circuit behavior regards the variations. Interval arithmetic presents a potential alternative to evaluate circuit designs under variations uncertainties.

In this work, we present simulation flow that utilize using of existing designs by replacing statistical parameters variations by interval parameters, so it may replace or enhance the current conventional Monte-Carlo simulation flow. An interval-value based circuit simulation engine is implemented, and library for interval models for sources, linear elements and non-linear elements.

Models library is tested for accuracy against Monte-Carlo simulations. Simulator is tested using linear elements circuits, showing acceptable results for small circuits.

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# Dedication

*To  
My Mother  
My Father  
and  
My Wife*



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