Ain Shams University
Faculty of Science

Reliability Analysis of Some Consecutive-k-out-of-n Systems in a Stress-Strength Setup

A Thesis

Submitted in Partial Fulfillment for
the Requirements of the Degree of Master of Science
in Mathematical Statistics

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Submitted to
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Cairo Egypt

2019

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Abbreviations and Notations

The abbreviations and notations we used adopted throughout the thesis.

1. Abbreviations:

We use the following abbreviations:

PDF	probability density function.
CDF	cumulative distribution function.
$i.i.d$	Independent and identically distributed.
MLE	maximum likelihood estimator.
EM	Expectation-Maximization algorithm.

2. Notations:

The following notations are used throughout the thesis.

n	Number of components of the system.
k	The minimum number of consecutive failed components which cause the system failure.
Regular linear	Linear consecutive k-out-of-n: F system, which is not relayed.
γ	$\gamma = r$ stands for a regular linear, $\gamma = u$ stands for a unipolar-relayed linear, $\gamma = b$ stands for a bipolar-relayed linear.
$\mathrm{R}_{\gamma}(n, k; p)$	Reliability of γ consecutive k-out-of-n: F system, with identical components reliability p.

$\underline{P}(a) = [p_1, p_2]$	Vector of reliabilities, where p_1 denotes the reliabilities of components from the first component in the system up to the component at position a (change point), and p_2 denotes the reliabilities of components from the component at position $a + 1$ up to the last component in the system.
$R_\gamma\left(n, k; c, \underline{P}(c)\right)$	The reliability of γ consecutive k-out-of-n: F system, with a change point, at position c, with components reliabilities $\underline{P}(c)$.
m	Number of change points in the system.
$\underline{a} = [a_1, \dots, a_m]$	Vector of the positions of the change points of the system.
$P_j(i) = [p_i, p_{i+1}, \dots, p_j]$	Vector of the reliabilities of the system components, the system consists of $j - i + 1$ components, first component with reliability p_i, second with reliability p_{i+1}, …, and last with reliability p_j.
$\underline{P}(\underline{a}) = [p_1, p_2, \dots, p_{m+1}]$	Vector of reliabilities, where p_1 denotes the reliabilities from the first component in the system up to the component at position a_1, p_2 denotes the reliabilities of components from the component at position $a_1 + 1$ up to a_2, …, and p_{m+1} denotes the reliabilities of components from the component at position $a_m + 1$ up to the last component in the system.
$R_\gamma\left(n, k; P_n(1)\right)$	The reliability of γ consecutive k-out-of-n: F system, for $2k \geq n$, with components reliabilities $P_n(1)$.

$R_\gamma\left(n,k,m;\underline{C},\underline{P}(\underline{C})\right)$	The reliability of γ consecutive k-out-of-n: F system, for $2k \geq n$, with m change points, at positions $\underline{C}$, and $(m+1)$ different reliabilities.

$$x_i = \begin{cases} 1 & \text{if the component at position } i \text{ is operating} \\ 0 & \text{if the component at position } i \text{ is failed} \end{cases}, i = 1,2,\dots,n.$$

δ_i	The length of the zero's run at i^{th} component.
$L_j^0(l)$	The longest zero's run in $x_l,\dots,x_j$, $l=1$, and $j=n$, or $l=2$, and $j=n,n-1$.
$R_{(s:s)\gamma}(n,k;c)$	Stress-strength reliability of γ consecutive k-out-of-n: F system, with a change point, at position c.
$R_{(s:s)\gamma}(n,k,m;\underline{C})$	Stress-strength reliability of γ consecutive k-out-of-n: F system, with m change points, at positions $\underline{C}$.

List of Publications

1. N. Mokhlis, M. Nassar, and S. Bakry. (2018). Stress strength reliability of regular and relayed linear consecutive k out of $n : F$ systems with m change points. *Journal of Statistics Applications & Probability*, **7**, 1-13.

2. S. Bakry and N. Mokhlis. (2019). Exact reliability formula for a linear consecutive k-out-of-n: F system and relayed consecutive systems with a change point for any $k \leq n$, with stress-strength application. *Pakistan Journal of Statistics and Operation Research*, **15**, 231-247.

Acknowledgements

I wish to express my thanks and deepest gratitude to ***prof. Dr. Nahed Abdel-Salam Mokhlis****, Emeritus Professor of Mathematical Statistics, Faculty of Science, Ain Shams University, for suggesting the objective of the work, helpful information, support, encouragement and especial assistance I have received from her through the valuable discussion, continuous interest, unfailing bits of advices and helping to get succeeded in my study. I am tremendously grateful for all the support and time she has given me over the research and writing up period. It would not be possible to complete this thesis without her endless support and understanding.*

Thanks and appreciations are also extended to ***prof. Dr. Manal Mohamed Nassar****, Emeritus Professor of Mathematical Statistics, Faculty of Science, Ain Shams University, and* ***Dr. Mohamed Hassan Abdel-Aziz****, Associate Professor of Scientific Computing, Faculty of Computer and Information Science, Ain Shams University, for their help and support during the research.*

My special thanks to ***Prof. Dr. Hassan Ramadan****, Chairman of the Department of Basic Science, Faculty of Computer and Information Science, Ain Shams University, for his continuous support.*

I am particularly pleased to thank Department of Mathematics, Faculty of Science, Ain Shams University. All my professors and demonstrators whom have taught and supported me.

A special thanks is given to my parents ***Dr. Mohamed Bakry*** *and* ***Mrs. Fatma El-Shemy****, my sisters* ***Amira****,* ***Nesreen*** *and my brother* ***Abdallah*** *for their love, prayers and continual support throughout my life. I would also like to thank my friends for their continuous encouragement.*

Abstract

The aim of this thesis is to study the reliability of some linear consecutive k-out-of-n: F systems, with a stress-strength setup. Explicit expressions for the reliability of regular and relayed linear consecutive k-out-of-n: F systems, with one change point are obtained for any value of k, $1 \leq k \leq n$, by conditioning on the state of the component at the change point, working or failed. A change point in a system means that the reliabilities of the components before and at this point differ from those after it. The case of having more than one change point is also discussed, and explicit expressions of the reliability are obtained for regular and relayed linear consecutive k-out-of-n: F systems, when $2k \geq n$, by using the longest zeros' run statistic. The components of the system are assumed to be independent. The change in reliabilities of the components may be due to change in the stress imposed on the components, change in the strength of the components, or change in both the stress and the strength. These three possibilities are studied, when there is one change point or when there are more than one change point. Formulas of the stress-strength reliabilities are derived generally for any continuous distributions for the stress and the strength. As application two cases are discussed: Case I, when the stress and the strength have distributions with the same exponential form, and Case II, when the stress and the strength have different distributions forms. In Case I explicit expressions for the stress-strength reliability of regular and relayed linear consecutive k-out-of-n: F systems are obtained under the three possibilities of a change point (one, or more). For Case II, as illustration the stress is assumed to have negative exponential distribution, while the strength is assumed to have generalized Lindley distribution. Estimation of the stress-strength