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Properties of Some Quantum Computing Models

A Thesis

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By

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In loving memory of the late Nobel Laureate in chemistry our founder Prof.
Ahmed Zewail (1946 – 2016). Until we meet again . . .

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Abstract

Quantum information Science offers the potential for major advances such as quantum computation which spare no efforts to offer the superior promise of performing mathematical and physical problems which are simply beyond the reach of any classical computer known at present. It has the potential to dramatically change the way we think about computation, programming and complexity.

Implementing quantum computers has been a complex mission, because of the challenges that faced the experimental implementation of quantum computation. In this thesis, we have taken a few small steps toward the implementation of a practical quantum computer, by introducing a new quantum computation implementation treatment, using the interaction between a system of superconducting qubits and a system of superconducting cavity.

The work presented in this thesis constitutes and develops a new approach for realizing quantum gates for the design of quantum algorithms. Also, we propose and analyze a detailed experimental procedure for implementing the quantum algorithms . Accordingly, this study is an initial attempt to investigate the relationship between the operation times of such gates while functioning in algorithm and the number of photons present in their resonant cavities.

The study leads to the properties of such systems from which Multi-qubit algorithms can be realized. In addition, the time for implementing the protocol using the proposed technique is compared to previous schemes, the comparison shows that the protocol can be successfully implemented within the present experimental limits. Also, the time required for operation is considerably less than the dephasing and relaxation times of the systems.

The use of the photon number as a main control in the realization of any algorithm is demonstrated. The results indicate the possibility of a full integration into the realization of multi-qubit multi-photon states and its application in algorithm designs. Furthermore, the technique that originated from this research may find use in other, perhaps more scalable quantum computer implementations in future experiments.

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