

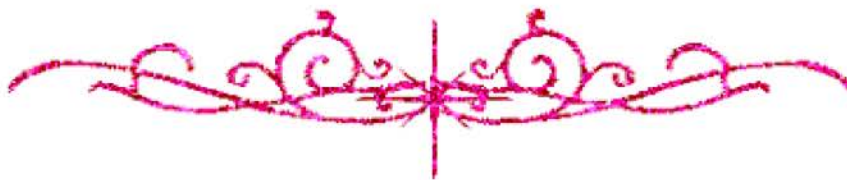
بسم الله الرحمن الرحيم



HOSSAM MAGHRABY



شبكة المعلومات الجامعية التوثيق الالكتروني والميكرو فيلم



HOSSAM MAGHRABY

جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

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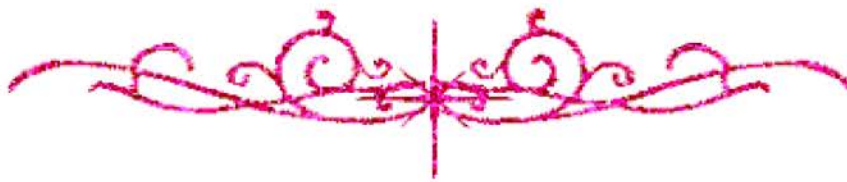
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HOSSAM MAGHRABY

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**Riemann and Lebesgue - Bochner
integrals of functions defined on a
non - void set X with values in a
linear convex space E**

Thesis

Submitted To mathematics department,

Faculty of Science,

Suez - Canal University

By

Essam Yassa Andrawis

B.Sc.

In Partial Fulfillment of the requirements for
M.Sc. Degree in Mathematics

Ismailia

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Approval Sheet

Title of thesis :

Riemann and Lebesgue - Bochner integrals of functions defined on a non-void set X with values in a linear convex space E .

Thesis Advisors

Approved

1- Prof. Dr. : Boulos B. Robiel

Professor of Mathematics ,

Faculty of Science ,

Cairo University

2- Prof. Dr. : Hamed M. Hawidi

Vice dean for higher studies and researches

Faculty of Science,

Suez Canal University

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Introduction

INTRODUCTION

In this thesis we study Riemann and Lebesgue - Bochner integrals of functions defined on a non - void set X with values in a linear convex space E .

This study has been done by many authors in the case of functions with values in Banach spaces [4], [6], [7], [8], [10].

The extension of the results from Banach spaces to linear convex spaces is not always straightforward and needs new ideas [2], [12].

This thesis consists of three chapters.

We give in the first chapter relevant background material needed for writing this thesis.

Chapter I consists of three sections. In section I.1. we give notations and results on measure spaces and step functions. In section I.2. we give some needed results on convergence of sequences in linear convex metric spaces. Finally, in section I.3. we give some adopted notations used throughout this thesis.

In chapter II we study Riemann integrals of functions defined on a segment $[a,b]$ with values in linear convex space. In section II.1 we give the classical theory of Riemann integral of real valued functions. In section II.2 we study Riemann integral of normed valued functions. We discuss mainly Lebesgue's criteria stating : A function $f : [a,b] \rightarrow \mathbb{R}$ is Riemann integrable iff it is bounded and

almost everywhere continuous. We give the example of Grave's [7] showing that in many Banach spaces the condition of the continuity almost everywhere is not necessary. We also give different proofs(see [7]) that the conditions of Lebesgue's criteria are sufficient for Riemann integrability in the case of normed valued functions. Also, we prove the beautiful result [10] showing that Lebesgue's criteria is true in the case of the functions $f : [a,b] \rightarrow l'$. In section II.3 we prove that the conditions of Lebesgue's criteria are also sufficient for Riemann integrability of functions with values in locally convex sequentially complete metric spaces.

Finally, in chapter III we survey results on Lebesgue - Bochner integral for functions defined on any non-void set X with values in a linear locally convex sequentially complete metric space E [2], [13]. In section III. 1 we give the definition of the space $L^1(\mu, E)$ and prove that $L^1(\mu, E)$ is a linear convex semimetric space. In section III .2 we prove the completeness of $L^1(\mu, E)$ and prove that it is also a locally convex metric sequentially complete space. In section III.3. we prove Lebesgue dominating theorem for $L^1(\mu, E)$.

Finally, in section III.4 we prove Fubini's theorem.

Chapter 1

Preliminary Results

CHAPTER I

PRELIMINARY RESULTS

In this chapter we give preliminary results used throughout this thesis.

In Section I.1 we give notations from measure theory. In section I.2 we give some needed results on convergence of sequences in locally convex spaces. Finally, In section I.3 we adopt notations used throughout this thesis.

I.1 Preliminary Notations.

In this section we give some notations and results used throughout this thesis . we begin by

DEFINITION I.1.1. let X be a nonvoid set , $\mathcal{A}$ be a semiring of subsets of X , let $\mu: \mathcal{A} \rightarrow \mathbb{R}$ be a σ -additive function. Then the triple $\{X, \mathcal{A}, \mu\}$ is called a measure space. Let $A_1, \dots, A_n \in \mathcal{A}$, let $A_i \cap A_j = \emptyset$, $i \neq j$, $i, j = 1, 2, \dots, n$ and let $X = \bigcup_{i=1}^n A_i$. Then $\{A_1, \dots, A_n\}$ is called a partition of X . Let E be a linear convex space over the field Σ ($\Sigma = \mathbb{R}$ or $\mathbb{C}$). We call the function $h: X \rightarrow E$ a step function if there is a partition $\{A_1, \dots, A_n\}$ of X and $\{a_1, \dots, a_n\}$ from E such that