



Cairo University

**HYBRID LEVEL SETS METHOD/FINITE ELEMENT FLOW  
SOLVER FOR VISCOUS FLOW SIMULATIONS WITH  
MOVING BOUNDARIES**

By

**Hossam Hassan Al-qaleiby Abo alela**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
In Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
In  
**Engineering Mechanics**

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
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**Title of Thesis:**

HYBRID LEVEL SETS METHOD/FINITE ELEMENT FLOW SOLVER FOR VISCOUS  
FLOW SIMULATIONS WITH MOVING BOUNDARIES

**Key Words:**

Moving boundary; Finite element method; level set method; Viscous flow;  
unsteady compressible flow.

**Summary:**

The Fluid-Structural interaction between the flow field and the burning surface of a solid propellant rocket is investigated. The burning surface evolution geometry is captured using the level sets method. The flow field is modelled using different levels of complexity namely starting from a lumped mass (zero-dimensional unsteady) model till two-dimensional axisymmetric unsteady compressible viscous model. The coupling is applied successfully to the problem of internal ballistics of a solid rocket motor. The results are validated against available experimental work. Also, the coupling is applied successfully to the problem of wedge flow with a moving edge and shocks interactions.

# Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

Name: Hossam Hassan Al-qaleiby Abo alela.

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Signature:

# Dedication

Dedicated to the memory of my father, Hassan Al-qaleiby Abo alela, who always believed in me. You are gone, but your support will never do.

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First, all gratitude is due to Almighty Allah for providing me the blessings to complete this work.

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*Hossam Hassan*

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