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بالرسالة صفحات

لم ترد بالأصل

CAIRO UNIVERSITY

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RELIABILITY ESTIMATION FOR TRUNCATED DISTRIBUTIONS

BY

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A Dissertation Submitted To The

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PREFACE

In most studies of systems reliability, the failure distributions of these systems are considered such that the lifetimes of these systems take values from zero to infinity. In many applications, we may note that these lifetimes have limits which are neither zero nor infinity. So, it would be more realistic in these applications if these lifetimes assumed not to be less or greater than a given age. It is necessary to state the difference between the truncated distributions, the translated distributions and the censored distributions. In truncated distributions, the random variable is assumed to take values in a specified interval such that any value is measured from the origin. For example, consider a system with lifetime ranging from A to B. This means that the minimum lifetime of the system is A (measured from the begining of time which is taken as zero) and the maximum lifetime is B (measured from the beginning of time which is taken as zero). In translated distributions, the random variable takes values in a specified interval also, but these values are not measured from the origin. They are measured from the point of translation. Thus when taking deviations of the random values from the translated point we shall have them as if it were measured from the origin. In other words, it turns to be a random variable with non translated distribution. The question of censored distributions arises when we are able to know the individual values of observations below (or above) a given value only. Suppose for example, that the distribution of lifetimes of an electric lamp is normal. To estimate this distribution we start an expriment where 100 lamps, say, are kept burning. After a certain time, for example, 1000 hours, we end the experiment and our observations now consist of two groups: (1) lamps for which we do not know the individual lifetimes but only that they exceed 1000 hours, and (2) lampls for which we know the individual lifetimes (which are less than 1000 hours). A distribution of this kind is called a censored distribution because the obtainable information in a sense has been censored [13].

In our study we shall be concerned with the reliability estimation for truncated lifetime distributions. In chapter I, we present the earlier studies which have been made in this field. In chapter II, the reliability estimation of one-unit system is given when the underlying lifetime distribution is above or doubly truncated. The points of truncation may be known or unknown. In cahpter III, we establish a minimum variance unbiased estimator of the reliability function of compound systems. The individual lifetime distribution is also assumed to be truncated from the right side or the left side or from both sides. The points of truncation may also be known or unknown. Special cases have readily been proved to be deduced from our results. In chapter IV we present a comparison between the maximum likelihood and minimum variance unbiased estimators of the reliability function of one-unit systems, when the lifetime distribution is assumed to be truncated from above at a known point of truncation.

Main parts of our study have been published (see [20], [21]).

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CHAPTER I.

INTRODUCTION.

In this chapter we present the earlier studies of reliability estimation for truncated exponential distributions. These studies depends on a result proved by Tukey [31] and Smith [28]. This chapter is divided into three sections. In section 1.1, we present the studies of both Tukey and Smith. In section 1.2 the work of Tate [30] is presented. Tate studied the estimation of functions of truncation parameters. In the third section, we present the study of Holla [15] and Varde and Sathe [25].

1.1. Truncation and sufficient statistics.

Generalizing the earlier remarks by Fisher and Hotelling, Tukey showed that if a family of distributions admits a set of sufficient statistics, then the family obtained by truncation to a fixed set or by a fixed selection ; also admits the same set of sufficient statistics. Tukey's proof assumed the relevant family of probability measures to be dominated by a fixed measure function and made use of the factorization theorem concerning sufficient statistics in this case. He presented the family of distributions in the form

$$d\,F(x/\theta) = C(\theta)\, f(x/\theta)\, d\mu(x) \qquad (1.1)$$

where x is a possibly multidimensional random variable, θ is a possibly multidimensional parameter, $C(\theta)$-the normalizing factor of the distribution function-is a positive real function of θ , $f(x/\theta)$ - the relative probability density- is a non-negative real function of x and θ , and $\mu(x)$ is a positive measure function. In this representation the natural and sufficient condition that $\{h_i(x)\}$ are a set of sufficient statistics for θ is the existence of functions $a_i(\theta)$ such that

$$\frac{\partial \log f(x/\theta)}{\partial \theta} = \sum_i a_i(\theta) h_i(x) \qquad (1.2)$$

To prove his result, Tukey supposed that the family $F(x/\theta)$ is truncated onto a Borel set E, this means that

$$\Pr\{x \in E_2 / F(x/\theta) \text{truncated to } E\} =$$

$$= \frac{\Pr\{x \in E \cap E_2 / F(x/\theta)\}}{\Pr\{x \in E / F(x/\theta)\}}.$$

If $\phi_E(x)$ is the characteristic function of E and if

$$k(\theta) = \Pr\{x \in E / F(x/\theta)\} = \int_E dF(x/\theta)$$

then the probability element of $F(x/\theta)$ truncated to E is

$$\frac{C(\theta)}{k(\theta)} f(x/\theta)\, \phi_E(x) d\mu(x) = C'(\theta) f(x/\theta) d\nu(x)$$