Faculty of Education
Department of Mathematics

A study and stability analysis of electrically conducting fluids with mass and heat transfer under the influence of the Hall currents

A Thesis
Submitted for the Partial Fulfillment of the Requirements for the Master's Degree in Teacher's Preparation of Science in "**Applied Mathematics"**

(Fluid Mechanics)

Submitted to:

Department of Mathematics, Faculty of Education, Ain Shams University
By

Yasmeen Mostafa Mohamed Abd El-Fattah
Demonstrator in
Department of Mathematics, Faculty of Education, Ain Shams University

Under the supervision of

Prof. Dr. Nabil Tawfek EL-dabe
Professor of Applied Mathematics
Faculty of Education - Ain Shams University

Prof. Dr. Galal Mahrous Moatimid
Professor of Applied Mathematics
Faculty of Education - Ain Shams University

Dr. Mona Abd El-hamed Ali
Associate professor of Applied Mathematics
Faculty of Education - Ain Shams University

(2019)

Faculty of Education

Department of Mathematics

Candidate: Yasmeen Mostafa Mohamed Abd El-Fattah

ThesisTitle: A Study and stability analysis of electrically Conducting fluids with mass and heat transfer under the influence of the Hall currents

Degree: Master Degree for Teacher's Preparation of Science in "**Applied Mathematics**"

(Fluid Mechanics)

Supervisors:

No	Name	Profession	Signature
1	**Prof. Dr. Nabil Tawfek EL-dabe**	Professor of Applied Mathematics Department of Mathematics Faculty of Education Ain Shams University	
2	**Prof. Dr. Galal Mahrous Moatimid**	Professor of Applied Mathematics Department of Mathematics Faculty of Education Ain Shams University	
3	**Dr. Mona Abd El-hamed Ali**	Associate professor of Applied Mathemtics Department of Mathematics Faculty of Education Ain Shams University	

Acknowledgement

First and foremost, my gratitude and thanks to gracious **Allah** who always helps and guides me. I would like to thank **the prophet Mohamed** "peace be upon him" who urges us to seek knowledge and who is the teacher of mankind.

I would like also to thank the supervision committee who are:

It's a great pride for me to get a chance to work with **Prof. Dr. Nabil Tawfek Mohamed El-dabe,** Professor of Applied Mathematics, Faculty of Education, Ain Shams University, for his suggestions for the research problems and his continuous encouragement and his support to me, his precious and infinite bids, his valuable ideas and suggestions that help me to go on in different stages of my research, his honest passion for work and science and his confidence which he gave to me encouraged me to work harder for deserving his trust , These were the key factors in my development as a person and a researcher. Allah gave him all good for what he did with me.

I am very grateful to **Prof. Dr. Galal Mahrous Moatimid** , Professor of Applied Mathematics, Faculty of Education, Ain Shams University, who helped me in this study through his suggestions for the research problems, his invaluable instructions, he teach me basics and ethics of writing scientific research. His helpful tips and continuous follow up in this study. He gave me much of his precious time and supported me with his wisdom, knowledge and experience through many discussions we had.

I am very thankful to **Dr. Mona Abd El-hamed Ali,** Associate professor of Applied Mathematics, Faculty of Education, Ain Shams University for her encouragement, valuable instructions, her guidance and providing me with valuable information and for her help during preparing the thesis.

Special great thanks and appreciation for go to **Prof. Dr. Mohamed Yahya AbuZeid Nassar,** Professor of Applied Mathematics, Faculty of Education, Ain Shams University for his kind cooperation and helpful tips while preparing the thesis. I am indebted to him for the time and attention he devoted to this project.

I also thank the Department of Mathematics - Faculty of Education - Ain Shams University represented by the head of the department, **Prof.** Dr**. Ehab Mohamed Fathy Abd El-Fataah** and my professors and my colleagues for their care and attention.

Finally, I can’t find words that express my gratitude and appreciation to my father, my mother, my grandmother my husband **Mahmoud**, my son **Yassen** and all my brothers for their support, help, patience, their invitations and continuous encouragement during my life. And I would like to thank my husband's parents and all my friends.

Yasmeen Mostafa Mohamed Abd El-Fattah
Cairo (2019)

Contents

A couple stress of peristaltic motion of Sutterby micropolar nanofluid inside a symmetric vertical channel with Hall currents effect

Effects of Hall currents on the peristaltic blood flow of Sisko micropolar nanofluid through a gap between two co-axial tubes (catheter) with non-Darcian porous medium

Summary

In this thesis, analytical and numerical solutions for some problems of the peristaltic transport were studied and some non-Newtonian models are taken into our account. The main aim of this thesis is to study the peristaltic transport, which is a form of fluid transport, under the influence of Hall currents effect. Also, the distributions of temperature, concentration of nanoparticles, spin velocity and mass concentration through peristaltic flow are taken into account in our studies. The analytical solutions of equations of motion are based on a Homotopy perturbation method. Meanwhile the numerical solutions of equations of motion are based on Runge - Kutta Merson with a shooting technique. The trapping phenomenon is also obtained. The numerical calculations with the help of graphs are adopted to obtain the influences of various parameters, such as the Hall current, magnetic field, Brownian motion parameters, Grashof, Eckert, Darcy, Biot, Schmidt, Soret, Dufour, and Prandtl numbers on the previous distributions.

This thesis consists of four chapters with their illustrated figures in addition to three appendices and list of references besides Arabic and English summaries. These chapters are outlined as follows:

Chapter 1

A presentation of the main items, which are included in the researches, is illustrated in this chapter:

- Classifications of fluids
- Peristaltic transport
- Heat and Mass transfer
- Hall currents effect
- Nanofluids
- Porous media
- Governing equations of fluid motion
- Homotopy perturbation method (HPM)

Chapter 2

In this chapter, the peristaltic transport of a non - Newtonian fluid obeying a Casson model with heat and mass transfer inside a vertical circular cylinder is studied. The considered system is affected by a strong horizontal uniform magnetic field together with the heat radiation and Hall currents. The problem is modulated mathematically by a system of partial differential equations that describe the basic behavior of the fluid motion. The boundary value problem is analytically solved with the appropriate boundary conditions in accordance with the special case, in the absence of the Eckert number. The solutions are obtained in terms of the modified Bessel function of the first kind. Again, in the general case, the system is solved by means of the Homotopy perturbation and then numerically through the Runge - Kutta Merson with a shooting technique. A comparison is done between these two methods. Therefore, the velocity, temperature, concentration and the volume flow rate distributions are obtained. A set of diagrams

are plotted to illustrate the influence of the various physical parameters in the forgoing distributions. Finally, the trapping phenomenon is also discussed.

The contents of this chapter have been accepted for publication in "Thermal Science." (2018)
Impact factor: 1.43

Chapter 3

In this chapter, we investigated the peristaltic transport of an incompressible micropolar non - Newtonian nanofluid following the Sutterby model. The heat and mass transfer inside the two-dimensional symmetric vertical channel is considered. The system is affected by a strong magnetic field together with thermal radiation, couple stress, chemical reaction, Joule heating, heat generation, Dufour & Soret and Hall currents effects. The governing equations of motion are analytically solved by utilizing the long wavelength and low Reynolds number approximations. Furthermore, the resulted boundary - value problem is solved by means of the Homotopy perturbation method (HPM). An illustration of the influence of the various physical parameters in the foreign distributions; such as Hall currents, magnetic field, Sutterby, couple stress, Brownian motion, thermophoresis and slip parameters, heat and mass transfer Biot numbers, Eckert and Dufour numbers are obtained throughout a set of graphs and tables. It is observed that the axial velocity enhances with an elevation in the Sutterby parameter. Whereas, it decreases with larger values of the magnetic field parameter. Furthermore, the temperature decreases with an enlarged in the value of a heat transfer Biot number. Whilst, the concentration enlarges with an increase in the values of mass transfer Biot numbers. Moreover, the trapping phenomenon is discussed throughout a set of figures. This depicts the variation of the streamlines under the impact of couple stress, amplitude ratio, and magnetic field parameters. It is noticed that the size of the trapped bolus increases with the increase in the foregoing three parameters. In addition, the number of the circulations increases with the elevation in the values of couple stress and amplitude ratio parameters, simultaneously; it has no changes with larger values of the magnetic field parameter.

Chapter 4

In this chapter, we investigated the peristaltic transport of an incompressible blood flow obeying the Sisko micropolar fluid and involving golden nanoparticles. The influence of heat and mass transfer between vertical coaxial tubes, at which the inner tube is considered as a rigid boundary. Simultaneously, the walls of the outer one are flexible and have a sinusoidal wave traveling down its wall. Strong magnetic fields, including Hall and ion slip currents, as well as sort, Dufour take into our consideration. The governing equations of motion are solved utilizing the approximations of the long wavelength and low Reynold's number. Moreover, the resultant equations are solved by means of the homotopy perturbation method (HPM). It is shown that the axial velocity magnifies with the increasing of the value of both Sisko and ion slip parameters. Whereas, the value of the axial velocity reduces with the increase in the value of the velocity slip parameter. On the other hand, the elevation in the value of Dufour number leads to an increase in the value of the temperature. Furthermore, the increasing of the value of the chemical reaction parameter makes a reduction in the value of nanoparticles concentration. Finally, the trapping phenomenon is illustrated throughout a set of graphs. It is found that the size of the trapped bolus enhances with the increment in the value of the magnetic parameter. In contrast, the size of the trapped bolus decreases with the increment in the value of the Sisko parameter.

List of publication

[1] Eldabe, N. T. M., Motiamid, G. M., Mohamed, M. A. A., Mohamed, Y. M., Effects of Hallcurrents with heat and mass transfer on the peristaltic transport of a Casson fluid through a porous medium in a vertical circular cylinder, has been accepted for publication in "**Thermal Science" (2018).**

[2] "A couple stress of peristaltic motion of Sutterby micropolar nanofluid inside a symmetric channel with a strong magnetic field and Hall currents effect" has been submitted for publication.

[3] "Effects of Hall Currents on the peristaltic Blood Flow of Sisko micropolar nanofluid inside Catheter through a Non-Darcian Porous Medium" has been submitted for publication.

Chapter 1

Introduction

1.1 Classifications of fluids

1.1.1 Newtonian and non-Newtonian fluids

A Newtonian fluid is a fluid whose stress versus strain rate relation is linear and passes through the origin. The constant of proportionality between stress and strain is known as the coefficient of viscosity. A simple equation to describe Newtonian fluid behavior is

$$\tau = -\mu \frac{\partial u}{\partial y} \tag{1.1}$$

where τ is the shear stress exerted by the fluid, μ is the coefficient of viscosity (a constant of proportionality), $\frac{\partial u}{\partial y}$ is the velocity gradient perpendicular to the direction of shear (rate of strain). When a fluid is sheared between a fixed plate and a moving plate, the coefficient of viscosity is given by the Eq.

$$\mu = \frac{\text{(shearing stress)}}{\text{(velocity gradient)}} = \frac{\text{(force/are a)}}{\text{(velocity/length)}}\,. \tag{1.2}$$

For a Newtonian fluid, the viscosity depends only on temperature and pressure and does not depend on the forces acting on it. Many of the fluids encountered in everyday life (such as water, air, gasoline, mineral oils, molten metals and honey) are adequately described as being Newtonian fluids [1].

In non-Newtonian fluids, the relation between the shear stress and the shear rate is non-linear. For non-Newtonian fluids, the viscosity can change by many orders of magnitude as the shear rate changes [2].

1.1.2. Different types of the non-Newtonian fluids

Non-Newtonian fluids can be classified as time independent fluids, time dependent fluids and viscoelastic fluids.

These types are summarized in the following table [2, 3, and 4].

Table 1.1: Types of non-Newtonian behavior.

Time independent fluids	Shear thickening (dilatant)	Apparent viscosity increases with the increase of stress:
	Shear thinning (pseudoplastic)	Apparent viscosity decreases with the Increase of stress:
	Generalized Newtonian fluids	The relation between stress and strain is linear:
Time dependent fluids	Rheopecty	Apparent viscosity increases with duration of stress:
	Thixotropic	Apparent viscosity decreases with duration of stress:
Viscoelastic fluids	Kelvin material and Maxwell	Both viscous and elastic properties have been possessed:

1.1.2.1. Time Independent Fluids [3]

The viscosity of time independent fluid is dependent on the shear rate. It has the following three types (as shown in Figure 1.1):

(i) **Shear thickening**

It is also called dilatant. The apparent viscosity increases with the increase of shear rate. Some examples of shear thickening fluids are sugar in water, sand in water, clay slurries and suspensions of corn starch. Shear thickening fluids are also used in all wheel drive systems utilizing a viscous coupling unit for power transmission.

(ii) **Shear thinning**

It is also called pseudo-plastic. The apparent viscosity decreases with increasing shear rate. Some examples of shear thinning are paper pulp in water, milk, gelatin, polymer, paints, ice, blood, syrup, molasses, some silicone oils and some silicone coatings.

(iii) **Visco-plastic fluids**

The behavior of this kind of non-Newtonian fluid is characterized by the existence of yield stress which must be exceeded for the fluid to deform (shear) or flow. Conversely, such a substance will behave like an elastic solid or like a rigid body when the externally applied stress is less than the yield stress. Bingham plastics are a special class of Visco-plastic fluids which have a linear shear stress or shear strain relationship which require a finite yield stress before they begin to flow (the plot of shear stress against shear strain does not pass through the origin). Several examples are clay suspensions, drilling mud, tooth-paste, mayonnaise, chocolate, and mustard. The surface of a Bingham plastic can hold peaks when it is still. By contrast Newtonian fluids have at featureless surfaces when still.

(iv) Generalized Newtonian fluids

A generalized Newtonian fluid is an idealized fluid for which the shear stress is a function of shear rate at the particular time, but not dependent upon the history of deformation. Although this type of fluid is non-Newtonian (i.e. non-linear) in nature, its constitutive equation is a generalized form of the Newtonian fluid. Some examples are blood plasma and custard.

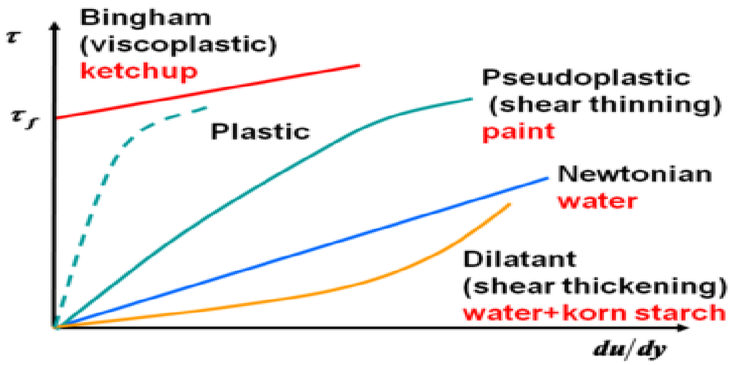

Figure 1.1: Flow curves of time independent fluids

1.1.2.2Time-dependent Fluids

It has the following two types (as shown in Figure 1.2):

(i) Rheopectic

In this type, effective viscosity increases with duration of shear rate. Rheopectic fluids are rare. Some examples are lubricants, whipped cream, pastes and printers inks.

(ii) Thixotropic

In this type, apparent viscosity decreases with duration of shear rate. Some examples are clays, some drilling mud, many paints and synovial.

1.1.2.3Viscoelastic Fluids

In this type of fluids both viscous and elastic properties have been possessing. The simplest type of such a material is one which is Newtonian in viscosity and obeys Hooks law for the plastic part. Viscoelastic fluids have not simple relationships between shear stress and shear rate, but it depends on the time derivatives of both of these quantities. Some examples are polymer melts, bread dough and egg white. Viscoelastic Fluids classify into two types of materials as follows:

(i) Kelvin material: linearity combination of elastic and viscous effects.

(ii)An-elastic: Material returns to a well-defined "rest shape".

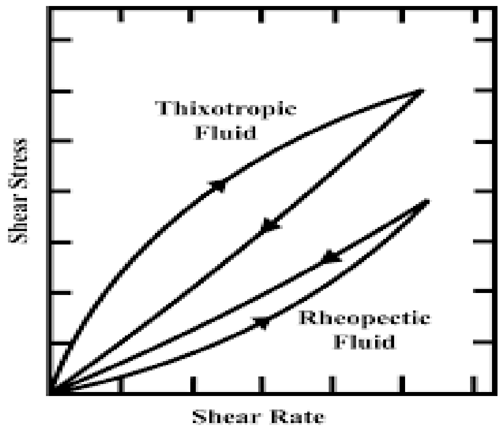

Figure 1.2: Flow curves of time dependent fluids

1.1.3 The constitutive equations for some models of the non-Newtonian fluids:

Recently, the interest in problems of non-Newtonian fluids has grown considerably because of the wide use of these fluids in many applications. These fluids are used in chemical process industries, food and construction engineering, petroleum production, power engineering and commercial applications. However, there is not a single governing equation which exhibits all the properties of non-Newtonian fluids. Rheological properties of materials are specified in general by their so-called constitutive equations. Some models of non-Newtonian fluids with their constitutive equations have been proposed in the following subsections.

1.1.3.1 Casson (Biviscosity) model

There is a great interest in the flow of non-Newtonian fluids through tubes possessing a definite yield value. These fluids used in polymer processing industries and bio-fluid dynamics. The most popular among these fluids is the Casson fluid. It has yield stress which no flow occurs. Casson model is used to describe the flow characteristics of blood. Casson model assumes the existence of the finite yield stress before the flow is existing. It leads to the plug flow. Also, it introduces a shear-dependent viscosity which consistent with the results of experiments of human blood [4].

The constitutive equation for this model can be written as [5]:

$$\tau_{ij} = \begin{cases} 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_\circ}}\right) e_{ij}, \pi_\circ \geq \pi_c \\ 2\left(\mu_B + \dfrac{p_y}{\sqrt{2\pi_c}}\right) e_{ij}, \pi_\circ < \pi_c \end{cases}, \tag{1.3}$$

where τ_{ij} is the stress tensor; p_y is the yielding stress; e_{ij} is the (i, j) component of the deformation rate; $\pi_\circ = e_{ij}e_{ij}$.

1.1.3.2 Sutterby model [6]:

The non-Newtonian material is very important due to its extensive applications in industrial, mechanical and biological processes. Pharmaceutical formulations, food items (gelatin, cornflour, jellies, paints, etc.) are few examples of practical implications of non-Newtonian materials. Non-Newtonian fluids are those materials which have nonlinear relation between shear stress and shear rate. One of these non-Newtonian fluids is the Sutterby fluid which shows relations for high polymer aqueous solutions. Sutterby fluid model has been in the limelight because of its much compact mathematical structure and the ability to demonstrate properties of pseudoplastic as well as dilatant fluids.

The constitutive equation for Sutterby fluid model may be written as [7]

$$\underline{S} = \frac{\mu}{2}\left[\frac{\sinh^{-1}(\Gamma\xi)}{\Gamma\xi}\right]^m \underline{A}_1, \tag{1.4}$$

$$\underline{A}_1 = (\nabla\underline{V}) + (\nabla\underline{V})^T, \tag{1.5}$$

and

$$\xi = \sqrt{\frac{1}{2}\left(tr\underline{A}_1^2\right)}. \tag{1.6}$$

$$\sinh^{-1}(\Gamma\xi) \cong \Gamma\xi - \frac{1}{2.3}(\Gamma\xi)^3 + \cdots, \quad |\Gamma\xi| < 1 \tag{1.7}$$

as

$$\sinh^{-1}(x) \cong x - \frac{1}{2.3}(x)^3 + \cdots. \quad |x| < 1 \tag{1.8}$$

where, $\underline{S}$ is the extra stress tensor for Sutterby model, Γ is the material constants for Suterrby model, $\underline{A}_1$ is the first Rivlin Erickson tensor and m is the power index,

1.1.3.3 Sisko model:

Sisko proposed a new model in studying the non-Newtonian fluid, which is later called Sisko fluid. Sisko fluid is a model which combines the features of viscous and generalized of power law models. It is capable of describing shear thinning and thickening phenomena, which commonly exist in nature for alternative values of (n). The fluid exhibits shear thinning properties when ($n<1$), the fluid shows Newtonian

behavior when ($n=1$) and when ($n>1$) the fluid shows shear thickening fluid [8]. Sisko fluid model has many industrial applications such as waterborne coatings, metallic automotive, cement slurries, lubricating greases, pseudoplastic fluids and drilling fluids [9]. Therefore, this model may consider as a blood model.
The constitutive equation can be written as [9]:

$$\underline{S} = \left[a + b_1 [\zeta]^{n-1} \right] \underline{A}, \tag{1.9}$$

$$\underline{A} = (\nabla \underline{V}) + (\nabla \underline{V})^T, \tag{1.10}$$

where $\underline{S}$ is the stress deviator, $\zeta = \frac{1}{2}\left(tr(\underline{A})^2 \right)$, $tr(\underline{A})^2$ is the sum of elements in main diagonal of $(\underline{A})^2$, n is the power index and a, b_1 are the material constant for Sisko fluid.

1.2 Peristaltic transport:

1.2.1Definition of the peristaltic transport

The mechanisms of fluid transport are found in several localities inside the human body; one mechanism that stands out is peristalsis. It means that the transport of fluids or solids through a tube without any need of an overall pressure difference. The term peristalsis stems from the Greek word peristaltikos, which means clasping and compressing (as shown in Fig. 1.3). The medical definition of peristalsis is "Successive waves of involuntary contraction passing along the walls of a hollow muscular structure and forcing the contents onward". Peristaltic mechanisms may be involved in swallowing of food through the esophagus, vasomotor of small blood vessels, spermatic flows in the ducts of the male reproductive tract, embryo transport in the uterus, among others. In much of the gastrointestinal tract, smooth muscles contract in sequence to produce a peristaltic wave which forces a ball of food (called a bolus while in the esophagus and gastrointestinal tract and chyme in the stomach) along the gastrointestinal tract. In the urinary system, the ureter, a tube which connects the kidney to the bladder, is one of those "hollow muscular structures" which pumps fluids by peristaltic motion [10].

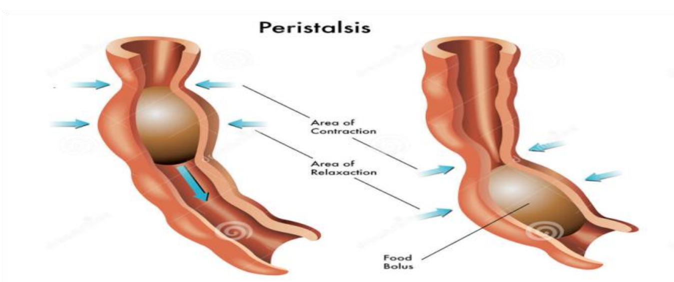

Figure 1.3: Peristalsis