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EFFECT OF TIME-ORDERING ON HADRON-NUCLEUS SCATTERING

A THESIS

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By

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ABSTRACT

In the study of particle-nucleus or nucleus-nucleus scattering in the framework of multiple scattering theory of Glauber, the commutativity of the profile function operators Γ_i is assumed. This approximation may ignore the actual situation in the multiple scattering processes. Therefore, in this work the noncommutative profile function operators are considered .The results of $p-d$, $p-^3He$ and $p-^4He$ elastic scattering differential cross sections at intermediate energies of order 1 GeV show that the consideration of noncommutativity of the profile function operators is important to obtain a good fit with the experimental data. This means that the time-ordering of collisions in multi-scattering processes must be considered.

SUMMARY

This thesis is concerned with the study of the effect of the time-ordering in multi-scattering processes in particle-nucleus elastic scattering at intermediate and high energy in the framework of multiple scattering theory of Glauber. The nuclei ^{2}H , ^{3}H , ^{3}He , and ^{4}He are taken as a targets. The time-ordering plays important role to obtain a good fit with the experimental data of particle-nucleus elastic scattering differential cross section at the minima. This effect is represented by a phase factor in particle-nucleon elastic scattering amplitude of second, third, and fourth collision and so on. Including the time-ordering in the calculations of the results of the particle-nucleus elastic scattering differential cross section, the agreement with the experimental data is improved at different minima for the mass number of the target nucleus A>2. The obtained results are, also, compared with the results of phase-variation of particle-nucleon elastic scattering amplitude suggested by Franco and Yin. The thesis consists of five chapters, list of references, list of figures, list of tables and English and Arabic summaries.

In chapter I, a general view of the subject in published literatures is presented for a wide range of years, 1959-2018, Sec. 1.1 particle-nucleon amplitude phase-variation is presented in Sec. 1.2. A brief formulation of multiple scattering theory of Glauber is given in Sec. 1.3. Finally, in Sec. 1.4 the time-ordering effect is introduced and explained.

Chapter II is concerned with the time-ordering effect on the elastic scattering differential cross section of proton-deuteron ($p-d$)

at 1 and 11.9 GeV, antiproton-deuteron ($\bar{p}-d$) at 1.271 GeV and π -meson-deuteron ($\pi^- - d$) at 8.862 GeV. At first, the mathematical formula of elastic scattering amplitude in the framework of Glauber multiple scattering theory is obtained, section 2.1.we take the D-state effect into account. In section 2.2 the results of calculations are presented and discussed. The results show that the effect of time-ordering is not clear. Therefore, we could not obtain a conclusion about the time-ordering effect in the case of hadron-^2H scattering.

In chapter III, we consider the effect of time – ordering of collision in the calculations of $p-^3He$, $p-^3H$ elastic scattering differential cross sections. The $p-^3He$, $p-^3H$ elastic scattering amplitudes are obtained in section 3.1. The results are obtained at the energies 0.6, 0.715 and 1 GeV and discussed, Sec. 3.2. Also, the results of Franco-Yin phase variation are presented for the same reactions and energies. We concluded that the inclusion of the time-ordering is important to obtain a good fit with the experimental data, especially, at the first and second minima where the Franco-Yin phase variation fails to fit the data at the two minima together.

In chapter IV we tried to confirm the previously obtained results concerning the importance of time-ordering effect in multi-scattering of proton with a nucleus. Therefore, $p-^4He$ elastic scattering at the energies 0.8, 1, 1.728 GeV is studied. In section 4.1 $p-^4He$ elastic scattering amplitude with time-ordering effect is obtained. The results of calculations of $p-^4He$ elastic scattering differential cross section at the considered energies are presented and discussed in section 4.2. Also, the results of

Franco-Yin phase variation are presented at the same energies. In conclusion, the consideration of noncommutativity of the profile function operators is important to obtain a good fit with experimental data. This means that the time-ordering of collisions in multi-scattering processes must be considered.

In chapter V, we explain the difference between the time-ordering approach and the phase variation of nucleon-nucleon amplitude suggested by Franco and Yin. The accuracy of the results is discussed by using χ^2-method. Finally, in conclusion, the importance of time-ordering in multi-scattering processes is confirmed.

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