



ENHANCING ACTIVE RADIAL DISTRIBUTION NETWORKS BY OPTIMAL SIZING AND PLACEMENT OF DG UNITS USING MODIFIED EVOLUTIONARY ALGORITHM

By

Mohamed Abdelbadea Abdelghany Hassan

A thesis submitted to the
Faculty of Engineering, Cairo University
In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

Electrical Power and Machines Engineering

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Under supervision of

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TITLE OF THESIS: ENHANCING ACTIVE RADIAL DISTRIBUTION NETWORKS BY OPTIMAL SIZING AND PLACEMENT OF DG UNITS USING MODIFIED EVOLUTIONARY ALGORITHM

Key Words: Crow Search Algorithm (CSA), Distributed generation (DG), Multiobjective Optimization, Sizing of DGs, Total Harmonic Distortion.

Summary:

In this thesis, two approaches are compared to get the optimal size and location of distributed generation units in radial distribution networks. The first approach is a single objective formulation which has the lowest power losses and makes an enhancement in the system voltage profile formulation with a voltage constraint of $\pm 5\%$. The second one is a multi-objective formulation, concentrates on minimizing power losses and also minimizing the voltage deviation, a weighted sum method is presented to create Pareto front and also to get the best compromise solution.

A new population-based metaheuristic optimization technique named Crow Search Algorithm (CSA), which is a nature-inspired algorithm based on the smart behavior of crows, is used. A modification on the evolutionary algorithm CSA is proposed using Gaussian and Cauchy density function has been applied on the two approaches to get the optimal size and location of DG.

Finally, the impact of adding DGs on system harmonics is investigated to ensure total harmonic distortion at all buses by simulating the whole radial distribution system using MATLAB /Simulink, where the installed DGs type is assumed inverter-based units. In case the limits are still ensured, the achieved solution is accepted. However, if the limits are violated, another solution is proposed for harmonics mitigation or cancellation.



DISCLAIMER

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references sections.

Name: Mohamed Abdelbadea Abdelghany Hassan	Date:
Signature:	

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LIST OF SYMBOLS AND ABBREVIATIONS

• Symbols

AP : Awareness probability

f(x) : Objective function

Fl : Flight length

I(l): The current flowing in line l

 I_i : Magnitude of branch current

 $I_{max}(l)$: Maximum current carrying capacity of line l

 L_B : Lower limit of the variable

 N_c : Flock size

No_b : The total number of buses

No_lines : The total number of branch lines

 $P_{D,i}^{with_DG}$: Total load active power with DG at bus i

 $P_{D.i}^{without_DG}$: Total load active power without DG at bus i

 $P_{DG.i}$: Active power injected by DG at bus i

 P_{DG_k} : Active power penetrated by k_{th} DG unit

 $P_{DG_{max}}$: Maximum allowed output active power of k_{th} DG unit

 P_{DG_min} : Minimum allowed output active power of k_{th} DG unit

 P_{Li+1} : Active loads that are connected at node i+1

 Q_{Li+1} : Reactive loads that are connected at node i+1

 P_{i+1} : Effective real power flows from node i+1

 Q_{i+1} : Effective reactive power flows from node i+1

 P_{load} : Total active load of the system

 $P_{loss(i)}$: Real power losses of branch j

 $Q_{loss(i)}$: Reactive power losses of branch j

 P_{supply} : Active power supplied by the grid as the original primary source

 $P_{T_DG_max}$: Total maximum allowed output of all added DG units (*N* units)

 $P_{T_DG_min}$: Total minimum allowed output of all added DG units (N units)

 $P_{T loss}$: The total power loss of the system

 $P_{T_loss_{without\ DG}}$: The total real power loss for the system without DG

 $P_{T_loss_{with DG}}$: The total real power loss for the system with DG

 $Q_{D.i}^{with_DG}$: Total load reactive power with DG at bus i

 $Q_{D.i}^{without_DG}$: Total load reactive power without DG at bus i

 $Q_{DG.i}$: Reactive power injected by DG at bus i

 R_i : Resistance of branch j

rand : Random variable (between 0-1)

r_i : Random number with uniform distribution (between 0-1)

 THD_{ν} : Voltage total harmonic distortion

 U_B : Upper limit of the variable

Vi : The voltage of i_{th} bus

Vmax : Maximum accepted voltage at any bus (1.05 pu)

Vmin : Minimum accepted voltage at any bus (0.95 pu)

 V_{rated} : The rated bus voltage of the network (1.0 pu)

 w_1 : Weighting factor 1

 w_2 : Weighting factor 2

 μ : The mean or expectation of the distribution

 σ^2 : The variance

 δ_i : The voltage angel at node i