



MODELLING AND CONTROL OF UNDERACTUATED SYSTEMS APPLIED TO A UAV WITH A CABLE-SUSPENDED LOAD

By

Mohamed Samir Mohamed Eldemerdash

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Aerospace Engineering

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Title of Thesis:

Modelling and Control of Underactuated Systems Applied to a UAV with a
Cable-Suspended Load

Key Words:

Quadcopters; Underactuated Mechanical Systems; Spherical Pendulum; Linear Quadratic Regulator; Unmanned Aerial Vehicle

Summary:

In this thesis, a nonlinear eight degree of freedom (8DOF) mathematical model was derived from the first principles for a quadcopter when its C.G. is shifted from its geometric centroid and when a cable-suspended load is hanged at a general position. The nonlinear model was then linearized using small disturbance theory. An adaptive controller based on Linear Quadratic Regulator (Adaptive LQR) was designed and simulated using the nonlinear model to stabilize the system and track a given trajectory while minimizing the swinging motion of the suspended load. Using MATLAB/SIMULINK, the designed controller showed good performance in both stabilizing the system and tracking different trajectories like circular, infinity-shaped, helical and rectangular trajectories.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

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Acknowledgements

All praise and thanks are due to the Almighty Allah who always guides me to the right path and has helped me to complete this thesis. There are many people whom I have to acknowledge for their support, help and encouragement during the journey of preparing this thesis. So, I will attempt to give them their due here, and I sincerely apologize for any omissions.

First and foremost, I would like to record my gratitude to my supervisors Prof. Gamal M. El Bayoumi and Dr. Osama S. Mohamady for their supervision, advice and guidance from the early stage of this research as well as giving me extraordinary experiences throughout the work. Above all and the most needed, they provided me unflinching encouragement and support in various ways. I am really indebted to them more than they know.

I wish to express my thanks and gratitude to my parents, the ones who can never ever be thanked enough, for the overwhelming love and care they bestow upon me, and who have supported me financially as well as morally and without whose proper guidance it would have been impossible for me to complete my higher education.

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