



Cairo University

STRENGTHENING OF RC SHORT COLUMNS WITH EXTERNAL CARBON FIBER POLYMERS

By

Mahmoud El.Sayed Mahmoud Mohammed Abo El.Kheir

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Structural Engineering

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Title of Thesis:

**STRENGTHENING OF R.C SHORT COLUMNS WITH
EXTERNAL CARBON FIBER POLYMERS**

Key Words:

Reinforced Concrete; carbon fiber; finite element models; cohesive zone model;
parametric study.

Summary:

This research investigate the behavior of strengthening normal reinforced concrete short columns using external FRP jacket. The main parameters are; load eccentricity (e/t), cross section aspect ratio, radius of corner and cross section, and effect of retrofit reinforced concrete columns partially.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

Name:

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Acknowledgment

In the name of Allah the Compassionate and the Merciful

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Nomenclature

A_s : Area of longitudinal steel reinforcement.
 A_{sp} : Area of transverse reinforcement.
 b : Breadth of the column section.
 E_1 : The slope of the linear first FRP stress strain model portion.
 E_2 : The slope of the linear second FRP stress strain model portion.
 E_c : Modulus of elasticity for concrete.
 E_{com} : The elastic modulus of the confinement in transverse direction.
 E_{FRP} : Modulus of elasticity for Fiber reinforcement.
 E_p : Plastic modulus for concrete.
 f'_{cc} : Confined concrete axial stress.
 f'_{co} : Unconfined concrete axial stress.
 f_{FRP} : Fiber reinforcement ultimate stress.
 f'_l : Confined concrete lateral stress.
 f_{yh} : Yield strength of transverse reinforcement.
 h : Depth of the column section.
 k_1 : Confinement efficiency constant.
 n : The shape parameter.
 r : Cross section corner radius.
 S : Pitch of spiral or circular hoop.
 t_{FRP} : Fiber reinforcement layer thickness.
 ϵ_{ao} : Ultimate axial strain of plain concrete.
 ϵ_{cc} : Confined concrete axial strain.
 ϵ_{co} : Unconfined concrete axial strain.
 ϵ_{FRP} : Ultimate tensile strain for Fiber reinforcement.
 ϵ_h : Fiber reinforcement strain in hoop direction.
 ϵ_l : Confined concrete lateral strain.
 ϵ_{ro} : The corresponding radial strain.
 γ_c : Poisson's ratio of plain concrete.
 θ_r : slope of the plastic branch in FRP stress strain model.