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BAYESIAN INFERENCE OF THE MIXTURE OF TWO LOMAX DISTRIBUTIONS AS A LIFETIME MODEL

Thesis

Submitted for a partial fulfillment of Ph.D. degree in Statistics

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2006

Acknowledgements

I would like to express my sincere gratitude and thanks to Dr. A.M.Nigm , Dr.S.A.Ismail and Dr.E.H.Elkhodary, for their kind supervision, valuable advices and encouragement during the preparation of this thesis. I am deeply indebted to my family.

TO MY MOTHER

Abstract

The problem of estimation and prediction in reliability and life testing theory has been considered by many authors using both Bayesian and classical approaches for various lifetime distributions and under different types of censoring. Loss functions play an important role in the Bayesian approach. In most of the past literature on mixtures of distributions, problems of estimation were considered under the assumption of the squared error loss function.

This thesis deals with Bayesian estimation of the parameters, the reliability function and the hazard rate of the mixture of two Lomax distributions with respect to three symmetric loss functions and two asymmetric ones. A study of the Bayesian estimation assuming the quadratic, the weighted and ElSayyad loss functions is given for the mixture of two Lomax distributions. The linear exponential loss function and the modified linex loss functions are considered as asymmetric loss functions. A comparison between the two types of loss functions is made using the values of the posterior risks and the absolute relative error. Interval Bayesian estimation of the parameters of the mixture of two Lomax distributions is considered. The interim analysis is considered for the mixture of two Lomax distributions as a predictive application on the hypothesis testing. This application can be used to take a decision about continuing in drawing a sample or not. Bayesian predictive intervals for a bivariate Lomax distribution are considered. The one-sample and two-sample prediction are proposed. Numerical examples are provided to illustrate the theoretical work.

Keywords:

-Mixture of two Lomax distributions

-Bayesian estimation and prediction

-Interim analysis

-Bivariate Lomax distribution

-type I censoring

-type II censoring

-Symmetric loss function

-Asymmetric loss function

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Summary: This thesis deals with Bayesian estimation of the parameters, the reliability function and the hazard rate of the mixture of Lomax distributions with respect to three symmetric loss functions and two two asymmetric ones. A study of the Bayesian estimation assuming the quadratic, the weighted and ElSayyad loss functions is given for the mixture of two Lomax distributions. The Linex loss function and the modified linex loss functions are considered as asymmetric loss functions. A comparison between the two types of loss functions is made using the values of the posterior risks and the absolute relative error. Interval Bayesian estimation of the parameters of the mixture of two Lomax distributions is considered. The interim analysis is considered for the mixture of two Lomax distributions as a predictive application on the hypothesis testing. This application can be used to take a decision about continuing in drawing a sample or not. Bayesian predictive intervals for a bivariate Lomax distribution are considered. The one-sample and two sample prediction are proposed. Numerical examples are provided to illustrate the theoretical work.

List of Abbreviations and Notations

θ	a parameter or a vector of parameters
θ^*	an estimator of a parameter or a vector of parameters
$\underline{X_n}$	$X_1, X_2, \ldots\ldots\ldots, X_n$
P_j	the mixing proportion of the j^{th} component
$f_j(x)$	the probability density function of the j^{th} component
n	the informative sample size
λ	the number of the mixed subpopulations.
$F(x\|\theta)$	the cumulative distribution function of X given θ
$R(t)$	the reliability function
$h(t)$	the hazard rate
$\underline{x_n}$	$x_1, x_2, \ldots\ldots\ldots, x_n$
$L(\underline{x}\|\theta)$	the likelihood function
$L(\theta^*,\theta)$	the loss function of estimating θ by θ^*
Δ	$\theta^*-\theta$
Δ_1	$\dfrac{\theta}{\theta^*}-1$
$\pi(\theta)$	the prior distribution of θ
Ω	the parameter space
S^*	the sample space
$\pi^*(\theta \mid \underline{x_n})$	the posterior density function of θ given the observations $\underline{x_n}$
m	the future sample size
$f(y_j \mid \underline{x_n})$	the predictive density function of Y_j given $\underline{x_n}$
pdf	the probability density function
cdf	the cumulative distribution function
BPI	the Bayesian predictive interval
L	the lower bound of the *BPI*
U	the upper bound of the *BPI*
$G(\theta)$	the cdf of the random variable θ
$\underline{X_m}$	$X_{n+1}, X_{n+2}, \ldots\ldots\ldots, X_{n+m}$

c^*	the scale parameter of the Lomax distribution function
a^*	the shape parameter of the Lomax distribution function
$P(II)(c^*,a^*)$	Pareto distribution of the second kind or the Lomax distribution
$f(x)$	the probability density function
a_i	the shape parameter of the i^{th} component of the Lomax distribution
c_i	the scale parameter of the i^{th} component of the Lomax distribution
r	the number of units which have failed during the interval $(0, t_0)$
x_{ij}	the failure time of the j^{th} unit belonging to the i^{th} subpopulation, $j=1,\ldots,r_i$, $i=1,2$
b,d	the two parameters of the beta distribution
β_i, γ_i	the two parameters of the gamma distribution
IP	the informative prior
NIP	the non informative prior
s	the minimum sample size in the interim analysis
$\beta(*,*)$	the Beta function $(*,*)$
$G(*,*)$	the Gamma function $(*,*)$
$in\beta_u(*,*)$	the incomplete Beta function
$inG_u(*,*)$	the incomplete Gamma function
k^*	the number of the multivariate random variables
$\lambda_\eta(x)$	a continuous monotone increasing and differential function of X
a,c	the parameters of the bivariate Lomax distribution
$\Im^*$	class of multivariate distributions
d_1	the decision of continuing the experiment
d_2	the decision of stopping the experiment
LINEX	the linear exponential loss function
MELO	the minimum expected loss function
MLE	the maximum likelihood estimator

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Introduction and Review of Literature

Finite mixtures of distributions have proved to be of considerable interest and importance in recent years. They are used as models in a variety of important situations where a non homogeneous population may be considered as comprising two or more components. Mixtures of distributions arise frequently in life testing, reliability, biological and physical sciences. Some of the most important references that discussed statistical properties, inferences and several practical applications of different types of mixtures of distributions are a monograph by Everitt and Hand (1981) and two survey books by Tittrington, Smith and Makov (1985) and McLachlan and Basford (1988). Review papers are presented by Nigm and Al Hussaini (1997) and Al Hussaini and Sultan (2001).

A random variable X is said to have a mixture density function $f(x)$ with λ components if :

$$f(x) = \sum_{j=1}^{\lambda} p_j f_j(x) \ ;$$

where $j=1, 2, 3, \ldots, \lambda$, p_j are nonnegative real numbers, known as the mixing proportion, such that $\sum_{j=1}^{\lambda} p_j = 1$ and $f_j(x)$ is the density function of the j^{th} component of the mixture. Mixtures of distributions are important in modeling the failure data. Here the observations are times to failure of a sample of items. Often, failure can occur for more than one reason and the failure distribution for each reason can be adequately approximated by a simple density function. The overall failure distribution is then a mixture. Several attempts have been made to fit such mixtures to the failure distribution of electronic valves. Mendenhall and Hader (1958) fit exponential components. Ashton (1971) has considered a mixture of a gamma distribution with an exponential distribution to model the frequency distribution of time gaps in road traffic. An excellent review for mixtures of distributions was presented in Al Hussaini and Sultan (2001).

Several authors have treated the Bayesian estimation of parameters of mixtures of various distributions. The Bayesian estimation of the two scale parameters, the mixing parameter and the reliability of the mixture of two exponential distributions has been presented by