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شبكة المعلومات الحامعية

# بسم الله الرحمن الرحيم



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شبكة العلومات الحامعية



شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم





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# جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

# قسو

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات



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تحفظ هذه الأقراص المدمجة يعيدا عن الغيار



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بالرسالة صفحات لم ترد بالأصل





Ain Shams University. Faculty of Education. Department of Mathematics.

## ON BIOMATHEMATICS OF ENDOLYMPH

#### **Thesis**

Submitted in Partial Fulfillment of the Requirements of the Doctor of Philosophy Degree in Teacher Preparation in Science (Pure Mathematics)

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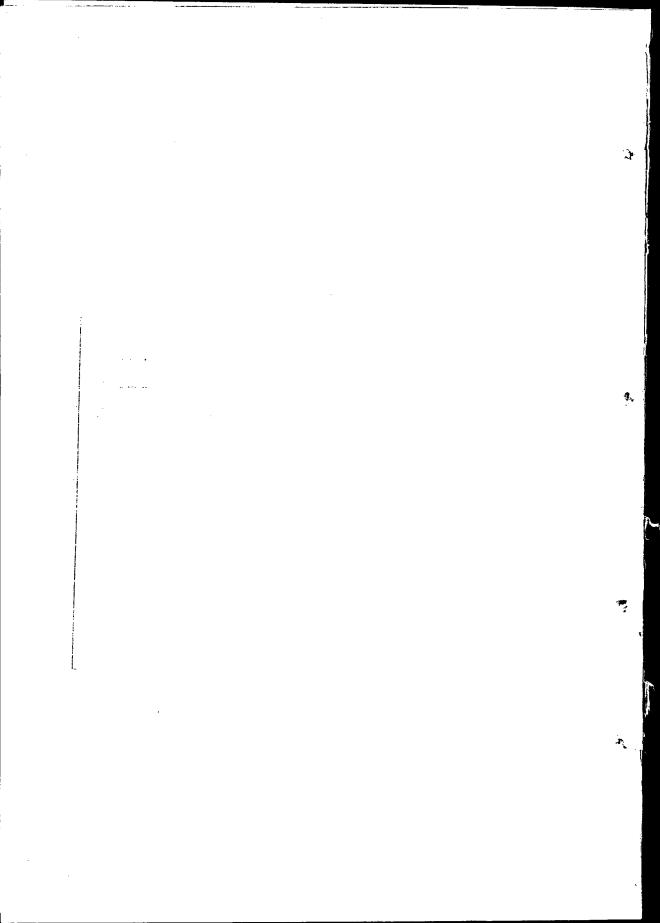
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Knowledge of the mechanics of the semicircular canals has already been useful in helping acrospace planners avoid situations which might prove discomforting to the astronaut.

This is an adequate reason alone for wishing to study the mechanics of the canals, but a further incentive comes from the fact that a full understanding of the mechanics of healthy canals may contribute to the development of methods of diagnosis and treatment of canals in diseased state.

The primary objective of these investigations is to study these phenomena mathmatically and to develop an improved analytical model for the fluid dynamic response of the semicircular canals and Otolith organs.

The thesis consists of six chapters and a list of references. The first chapter is an introductory one. It exhibits the anatomy and physiology of the inner ear and also a brief summary of Meniere's disease, fractional calculus and flow in curved pipe.

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In the second chapter, an exact solution for the Endolymph equation has been obtained. The problem is analyzed using the constitutive application of the Laplace and finite Hankel transformation. It has been proved that the previous work of [Van Buskirk et al. 1987] is a special case of the results we have obtained.

Chapter three presents the response of the semicircular canal to several changes in skull veolcity. The mathematical modeling of cupula using fractional calculus has been used to represent the response of the semicircular canal.

In chapter four the mathematical modeling of Otolith-Organs has been investigated. They were modeled mathematically as a three-element system consisting

of viscous Endolymph fluid in contact with rigid otoconial layer that is attached to the skull by gel layer. The gel layer is considered as an elastico-viscous fluid. For a step change in skull velocity an analytical solution has been obtained using Laplace transformations.

Chapter five is devoted to the problem of the flow of Endolymph under the effect of a transverse magnetic field. In this chapter, Magnetohydrodynamic (MHD) flow of the Endolymph through a porous medium has been studied. With the help of integral transforms, an exact solution is obtained for the unsteady flow of the Endolymph as an electrical conducting fluid in the presence of an imposed transverse magnetic field. The velocity profile and the magnetic field strength are determined in the exact form. The solution obtained is exact and valid for all Hartmann, Reynolds and magnetic Reynolds numbers. Sensitivity and mean velocity have also been obtained. It has been proved that the previous works of Pathak and Gupta are special cases of the results we have obtained.

The last chapter, consists of three parts. In the first part, pulsatile flow of blood through a porous medium under the influence of body acceleration is studied. With the help of Laplace and finite Hankel transforms, analytical expressions for axial velocity, fluid acceleration, flow rate and shear stress have been obtained. It has been proved that the previous works of Chaturani and Palanisawy [1991] is a special case of the results we have obtained. In the second part pulsatile flow of blood through a porous medium, with entrance effect is studied. Analytical expressions for axial velocity have been obtained.

Several limiting cases of physical and biological importance are obtained. Part three presents the case, when the tube is subjected to a stochastic series of pulses, imposed on the boundary and acting in an axial direction. The conditions of flow stoppage have been obtained.

#### **CHAPTER 1**

## **INTRODUCTION**

