

***“3-D Forward Modeling and Inversion of  
Geo-magnetic Data Using Gauss Legendre  
Quadrature and Correlation Tomography  
Methods”***

***By***

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# APPROVAL SHEET

## TITLE

**"3-D Forward Modeling and Inversion of Geo-magnetic Data Using Gauss Legendre Quadrature and Correlation Tomography Methods"**

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