



Ain Shams University  
University College for Women  
(Arts, Science, and Education)  
Department of Mathematics

# **The Transfer of some Algebraic Properties from Rings and Modules into Their Extensions**

A THESIS  
SUBMITTED IN PARTIAL FULFILLMENT OF REQUIREMENTS  
FOR THE DEGREE OF  
MASTER OF SCIENCE (M.Sc.)  
(PURE MATHEMATICS)

**BY**

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Department of Mathematics  
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**( 2020 )**





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M.Sc. Thesis  
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Title of Thesis:

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Thesis supervisors:

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## COURSES

The student has passed the following courses in partial fulfillment of requirement for M. Sc. degree:

- |  |             |
|--|-------------|
| 1. Advanced Abstract Algebra.            | 3h per week |
| 2. Numbers Theory.                       | 3h per week |
| 3. Numerical Analysis.                   | 3h per week |
| 4. Writing Scientific Research.          | 3h per week |
| 5. Partial Differential Equations.       | 3h per week |
| 6. Differential Geometry.                | 3h per week |
| 7. Spectral Theory and its Applications. | 3h per week |
| 8. Ethics of Scientific Research.        | 3h per week |

Head of Mathematics Department









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Asmaa Nasser.





# Abstract

It was shown by Galovich that if  $R$  is a commutative unique factorization ring (UFR) with identity, then  $R$  is a local ring with a nil maximal ideal. In this thesis, we generalize Galovich's results to the non-commutative case. Also, we generalize Chon's results to the non-commutative case, we show that if  $R$  satisfies ACC on principal right (left) ideals, then  $R$  is atomic. Moreover, we study UFR by using lattice structure.

Keywords : UFR; symmetric ring; local ring.



