



Cairo University

**RAY TRACE EXAMINATION OF THE HUMAN EYE LENS UNDER
CONSTANT VOLUME ACCOMMODATION USING COMPUTER
MODELING**

By

Amany Amr Mohammed ELZawahry

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Biomedical Engineering and Systems

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
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Title of Thesis:

Ray Trace Examination of The Human Eye Lens Under Constant Volume Accommodation Using Computer Modeling

Key Words:

Schematic eye model; Exact ray tracing; Accommodation under constant volume of the human eye lens

Summary:

The present work aims at introducing a computer model that reconstructs the visual system of the human eye. The model considers that the lens is composed of a core and peripheral layers of different indices of refraction and curvatures. The adjustment of the dimensions of the lens is assumed to take place under constant volume conditions.

The model employs a ray tracing technique, so that it follows the incident rays to its final destination on the retina.

The results showed that the received image on the retina takes the same curvature, such that the curved images is highly focused.

Disclaimer

I hereby declare that this thesis is my own original work and that no part of it has been submitted for a degree qualification at any other university or institute.

I further declare that I have appropriately acknowledged all sources used and have cited them in the references section.

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Dedication

To my Grandfather, the one who kept praying and supporting me all the years. For loving me for who I am and for the way his eye sparkled with warmth and care while telling me “my dear” and for believing in me and my choices. May Allah grant you paradise.

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Nomenclature

A: Accommodation level

D: Diopters (reciprocal of focal length measured in meters)

F, F': Primary (front) and Secondary (Back) Focal points

HOAs: High Order Aberrations

IOL: Intraocular Lens

LSA: Longitudinal Spherical Aberration

N, N' : Front and Back Nodal points

P, P': Primary and Secondary Principal points

RMS: Root Mean Square

TSA: Transversal Spherical Aberration

V, V': Front and Back Vertex points