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MULTIPLE-VALUED LOGIC OF TIED IMPLICATIONS AND ITS USE IN FUZZY LOGIC

BY

MOATAZ SALEH MOHAMED EL-ZEKEY

A Thesis Submitted to the
Faculty of Engineering at Cairo University
In Partial Fulfillment of the
Requirements for the Degree of

DOCTOR OF PHILOSOPHY
IN
ENGINEERING MATHEMATICS

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FACULTY OF ENGINEERING AT CAIRO UNIVERSITY
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