

سامية محمد مصطفى



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سامية محمد مصطفى



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شبكة المعلومات الجامعية التوثيق الالكتروني والميكروفيلم



سامية محمد مصطفى



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جامعة عين شمس

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Zagazig University
Faculty of Science, Benha
Department of Mathematics



Fuzzy Neighbourhood Spaces

Thesis submitted

by

Galal Mokhtar Elsalamony

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Supervisors

Prof. Dr. Farouk M. El-Batanony

Department of Mathematics, Faculty of Science, Benha

Prof. Dr. Robert Lowen

Department of Mathematics, University of Antwerp, Belgium

Prof. Dr. Nehad N. Morsi

Department of Mathematics, Military Technical College, Cairo

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بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

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Introduction

Introduction

Since the introduction of fuzzy neighbourhood spaces, in the sense of [25], many papers have appeared analyzing and making use of this notion. Some papers showed the distinguished internal characterizations of the category of fuzzy neighbourhood spaces, denoted by FNS, for example [37, 48, 49].

The fuzzy topological spaces associated with the fuzzy uniform spaces in the sense of [24] and with the fuzzy proximity spaces, in the sense of [3], were shown to be fuzzy neighbourhood spaces [25, 38]. Subsequently, the notion of fuzzy syntopogeny as a unified theory of fuzzy neighbourhood spaces, Lowen fuzzy uniformities and Artico-Moresco fuzzy proximities was introduced by Katsaras [16] and Mashhour et al. [35].

Given a lower $\ast$ semicontinuous triangular norm T , Morsi [42, 43] introduced the two categories of fuzzy T -locality spaces, and of fuzzy T -neighbourhood spaces, denoted by T -FLS and T -FNS, respectively. In the special case $T = \text{Min}$, the categories T -FLS and T -FNS coincide with the categories of fuzzy neighbourhood-base spaces, denoted by FNBS [39] and FNS [25].

The fuzzy topological properties which are generalizations of topological concepts such as separation or compactness are studied extensively, but what about the properties which are purely fuzzy, in the sense that there is no classical counterpart for them. Examples of such properties are, being a fuzzy-neighbourhood space, being maximal [48], being a fuzzy neighbourhood base-space, and so on. Surprisingly, several of these natural properties gave rise to a

subconstructs which are bireflectively or bicoreflectively, or simultaneously bireflectively and bicoreflectively, embedded in the category of fuzzy topological spaces, denoted by FTS. The last situation is unusual, since most familiar topological constructs, such as the category of topological spaces, denoted by TOP, do not have non-trivial simultaneously bireflective and bicoreflective subconstructs. This is a phenomenon which sharply distinguishes FTS from TOP and other topological constructs, on the categorical level. R. Lowen and P. Wuyts [31] gave a criterion expressing stability for certain compositions of maps, by which the task of classifying these simultaneously bireflective and bicoreflective subconstructs became easier.

We now give a brief overview of the contents.

Chapter 1 provides the fuzzy topological background needed in the sequel. We list some properties of the residuation implication $\mathcal{I}_T$ associated with a lower semicontinuous triangular norm T . The main notions and results associated with T-FNS, due to [12,43], are also given.

In *Chapter 2*, we identify a property that holds in all the special types of interest from Katsaras' fuzzy topogenous orders, and we call it structure A . We then establish natural bijections between the family of saturated N . topogenous orders on a set X [35] and the family of fuzzy topogenous orders of structure A , considered as fuzzy relations on the ordinary power set of X . The results of this chapter are published in [16].

Chapter 3 summarizes the main definitions and results given in [42, 43] concerning the category T-FLS, including some interrelations of T-FNS and T-FLS are discussed.

Chapter 4 contains some basic ideas of category theory and of categorical topology. While *Chapter 5* surveys the main ideas and results of stable

subconstructs of FTS, as shown in [31,32]. Moreover, the total fuzzy topologies $\Omega(T\text{-FLS})$ and $\Omega(T\text{-FNS})$ are determined as given in [44].

In *Chapter 6*, we examine Sierpinski objects and their epireflective hulls in stable subconstructs of FTS. The fuzzy interior operators of the total fuzzy topologies $\Omega(T\text{-FNS})$ and $\Omega(T\text{-FLS})$ are calculated. Finally, we give some examples of stable subconstructs of FTS containing TOP, together with some internal properties. Some of these results are published in [7].

The results of Chapter 2 and of Chapter 6 are completely, done by the author.

Chapter (1)

Fuzzy T-Neighbourhood Spaces