بسم الله الرحمن الرحيم

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تم رفع هذه الرسالة بواسطة / حسام الدين محمد مغربي

بقسم التوثيق الإلكتروني بمركز الشبكات وتكنولوجيا المعلومات دون أدنى مسئولية عن محتوى هذه الرسالة.

ملاحظات:
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Ain Shams University
Faculty of Education
Department of Mathematics

Nonlinear Delay Harmonic Oscillator Using Perturbation Analysis

A Thesis
Submitted in Partial Fulfillment of the Requirements of the Master's Degree in

Teacher's Preparation of Science in Applied Mathematics

(Applied mathematics)

Submitted to:

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(2021)


وَمَا تَوْفِيقِيٓ إِلَّا بِٱللَّهِۚ
عَلَيْهِ تَوَكَّلْتُ وَإِلَيْهِ أُنِيبُ

Faculty of Education
Mathematics Department

<u>Candidate</u>: Yomna Yasser Hassan Mohamed Ellabban

<u>Thesis Title</u>: Nonlinear Delay Harmonic Oscillator Using Perturbation Analysis

<u>Degree</u>: Master for Teacher's Preparation of Science in Applied Mathematics

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Acknowledgements

I would like to thank ***ALLAH*** for helping and supporting me to achieve this work, and I hope from his mighty to accept my efforts to do my duty. I would like also to thank the supervision committee:

Thanks go to ***Prof. Dr. Fawzy Mohamed Fahmy Elsabaa***, Professor of Applied Mathematics, Faculty of Education, Ain Shams University, for accepting supervising my thesis, teaching me scientific research ethics and for his support, encouragement, and continuing guidance during preparing this thesis.

My deep gratitude also goes to ***Prof. Dr. Galal Mahrous Moatimid***, Professor of Applied Mathematics, Faculty of Education, Ain Shams University, who has suggested the topic of the thesis and guided me with continuous encouragement during the process of writing and finalizing it. He offered me much of his precious time, wisdom, and knowledge in every scientific consultation.

My deep gratitude also goes to ***Dr. Eng. Nasser Abdul-Fadeel Abdul-Hameed Saeed***, Assistant Professor of Engineering Mathematics, Faculty of Electronic Engineering, Menoufia University, who helped me at the first step in this study through his suggestions for the research problems and for his great efforts, precious time, wisdom, and knowledge in every scientific consultation, and his assistance in publishing research papers in prestigious scientific journals. He provided me with his encouragement and continuing guidance during preparing this thesis.

It is also my pleasure to thank ***Prof. Dr. Ehab Fathy Mohamed***, Head of Mathematics Department, Faculty of Education, Ain Shams University, and all staff members for providing me with all facilities required to the success of this thesis.

My great appreciation goes to my ***kind parents, brother, sister and my beloved son*** for their support, patience, sacrifice and continuous encouragement.

Yomna Y. Ellabban

Abstract

In this thesis, a mathematical study for mechanical vibrations of some dynamical systems in the presence of time-delay in some dynamical systems described by nonlinear ordinary differential equations.

In chapter one, we stated a literature review, and some of work done in this field.

In chapter two, we present comprehensive study for resonance cases of nonlinear dynamical system. Time-delayed position-velocity controller is proposed to suppress the considered system nonlinear vibrations. The acquired analytical results revealed that the loop-delay has a great influence on the controller efficiency.

In chapter three, different time-delayed controllers are introduced to explore their efficiencies in suppressing the nonlinear oscillations of a parametrically excited system.

In chapter four, a study for nonlinear integral resonant controller is presented that is used to suppress the vibration amplitude of a nonlinear dynamic model at parametrically excited system.

Summary

The main objective of this thesis is a mathematical study for mechanical vibrations of some dynamical systems in the presence of time-delay in some dynamical systems described by nonlinear ordinary differential equations. We studied the mechanical vibrations of the cantilever beam with time-delay. Different control algorithms were proposed to suppress these nonlinear vibrations. All different mathematical models were analyzed analytically applying homotopy multiple scales perturbation method. The acquired analytical results were validated numerically utilizing the appropriate standard Matlab solvers. Finally, a list of references regarding this discipline was cited, where the thesis is outlined as follows:

Chapter 1: This chapter is concerned with introducing the background necessary to understand the mechanical vibration problem, identifying time delay in the control system, some of important researches that deal with the nonlinear vibrations of the cantilever beam, time-delay, and the study of different methods of perturbation.

Chapter 2: The nonlinear transversal oscillations of a cantilever beam system at primary, superharmonic, and subharmonic resonance cases are investigated within this chapter. Time-delayed position-velocity controller is proposed to suppress the considered system nonlinear vibrations. The multiple scales homotopy approach is employed to analyze the controlled nonlinear model. The amplitude-phase modulating equations that govern the system dynamics at the different resonance cases are extracted. The stability charts of the loop-delay are obtained. The influence of the different controller parameters on the system vibration behaviors is explored. The acquired analytical results revealed that the loop-delay has a great influence on the controller efficiency. Accordingly, the optimal values of the loop-delay are reported and utilized to enhance the applied controller performance. Finally, numerical validations of the accomplished analytical results are performed, which illustrated an excellent agreement with the obtained analytical ones.

Chapter 3: In this chapter, Six different time-delayed controllers are introduced to explore their efficiencies in suppressing the nonlinear oscillations of a parametrically excited system. The

applied control techniques are the linear and nonlinear versions of the position, velocity, and acceleration of the considered system. The time-delay of the closed-loop control system is included in the proposed model. As the model under consideration is a nonlinear time-delayed dynamical system, the multiple scales homotopy method is utilized to derive two nonlinear algebraic equations that govern the vibration amplitude and the corresponding phase angle of the controlled system. Based on the obtained algebraic equations, the stability charts of the loop-delays are plotted. The influence of both the control gains and loop-delays on the steady-state vibration amplitude is examined. The obtained results illustrated that the loop-delays can play a dominant role in either improving the control efficiency or destabilizing the controlled system. Accordingly, two simple objective functions are introduced in order to design the optimum values of the control gains and loop-delays in such a way that improves the controllers' efficiency and increases the system robustness against instability. The efficiency of the proposed six controllers in mitigating the system vibrations is compared. It is found that the cubic-acceleration feedback

controller is the most efficient in suppressing the system vibrations, while the cubic-velocity feedback controller is the best in bifurcation control when the loop-delay is neglected. However, the analytical and numerical investigations confirmed that the cubic-acceleration controller is the best either in vibration suppression or bifurcation control when the optimal time-delay is considered. It is worth mentioning that this may be the first article that has been dedicated to introducing an objective function to optimize the control gains and loop-delays of nonlinear time-delayed feedback controllers.

Chapter 4: In this chapter, a nonlinear integral resonant controller is utilized for the first time to suppress the principal parametric excitation of a nonlinear dynamical system. The whole system is modeled as a second-order nonlinear differential equation (i.e., main system) coupled to a nonlinear first-order differential equation (i.e., controller). The control loop time-delays are included in the studied model. The multiple scales homotopy approach is employed to obtain an approximate solution for the proposed time-delayed dynamical system. The nonlinear algebraic equation that governs the steady-state oscillatxion amplitude has been extracted. The effects of the time-delays, control gain, and feedback gains on the performance of the suggested controller

have been investigated. The obtained results indicated that the controller performance depends on the product of the control and feedback signal gains as well as the sum of the time-delays in the control loop. Accordingly, two simple objective functions have been derived to design the optimum values of the loop-delays, control gain, and feedback gains in such a way that enhances the efficiency of the proposed controller. The analytical and numerical simulations illustrated that the proposed controller could eliminate the system vibrations effectively at specific values of the control and feedback signal gains. In addition, the selection method of the loop-delays that either enhances the control performance or destabilizes the system motion has been explained in detail.

Contents

Chapter 1

Introduction

Introduction

1.1 Motivation and background

The oscillatory motion in our life is everywhere, like heartbeats, lungs have oscillatory motion, when we walk our arms, legs they oscillate, the earth also oscillates during the earthquake. In general, vibration is an oscillatory motion, and it means a motion that repeats itself after an interval of time. Repeated motion due to vibration cause problems and may deform the systems. Depending on its magnitude, vibration can cause a significant amount of damage to the systems as it affects all kinds of engineering structures. Leissa and Qatu (2011) mentioned that due to the vibration there could be rapid wear of certain components like bearings and gears that results in a generation of noise. The oscillatory system can be treated as being either linear or nonlinear, wherein linear system the principle of superposition holds, and mathematical technique are available for their analysis. But, in the nonlinear system, the principle of superposition doesn't hold and the techniques for the analysis are still under development and difficult to apply. All the systems are basically nonlinear, the only assumption is that when the oscillations are small then we can consider them as linear. Accordingly, linearization is some kind of approximation to analyze nonlinear dynamical systems.

Kelly (2011) referred to vibration that is classified into two types that are free and forced vibration. In free vibration, if we disturb the system and let it allow to oscillate its own without applying any external force to the system, because of its inherent forces which is elastic force, damping force, and inertia force it oscillates its own. On the other hand, forced vibrations take place under the excitation of external forces. These excitation forces may be classified as being periodic, non-periodic, or random. Forced vibrations occur at the excitation frequencies, and it is important to note that these frequencies are arbitrary and therefore independent of the natural frequencies of the system. If the frequency of excitation coincides with one of the natural frequencies of the system, a condition of resonance is encountered and dangerously large oscillations may result that causes a failure of major structures (i.e. buildings, bridges, airplane wings…etc.).

1.2 Control system

Suppressing or controlling vibration becomes a vital, economic, and health issue. For vibration reduction in machines, buildings, and structures different forms of vibration control have been proposed. Vibration control is broadly divided into two categories which are passive and active control. Active vibration control is a technique for reducing vibration by using some kind of sensor to measure the motion, force, acceleration, or any other parameter of thing that is vibrating and powered actuator to generate a force to resist the vibrational motion. On the other hand, passive vibration control is the technique in which there is no use of the sensors or actuators and does not consume any power. Soong and Costantinou (2014) introduced different forms of active and passive vibration control techniques for vibration reduction in machines, structures, and buildings. An active control system consists of basic configurations that are (see Fig. 1.1):

- **a)** Sensors to measure structural response variables or external excitations, or both
- **b)** Devices to process the measured information and to compute necessary control forces needed based on a given control algorithm
- **c)** Actuators that usually powered by external energy sources to produce the required forces.

Maccari (2001) mentioned that when only the structural response variables are measured, the control configuration is referred to as closed-loop control since the structural response is continually monitored and this information is used to make continual corrections to the applied control forces. An open-loop control results when the control forces are regulated only by the measured excitations. In the case where the information on both the response quantities and excitation is utilized for control design, the term open-closed loop control is used. Dukkipati (2009) said that closed-loop systems use feedback where a portion of the output signal is fed back to the input to reduce errors and improve stability.

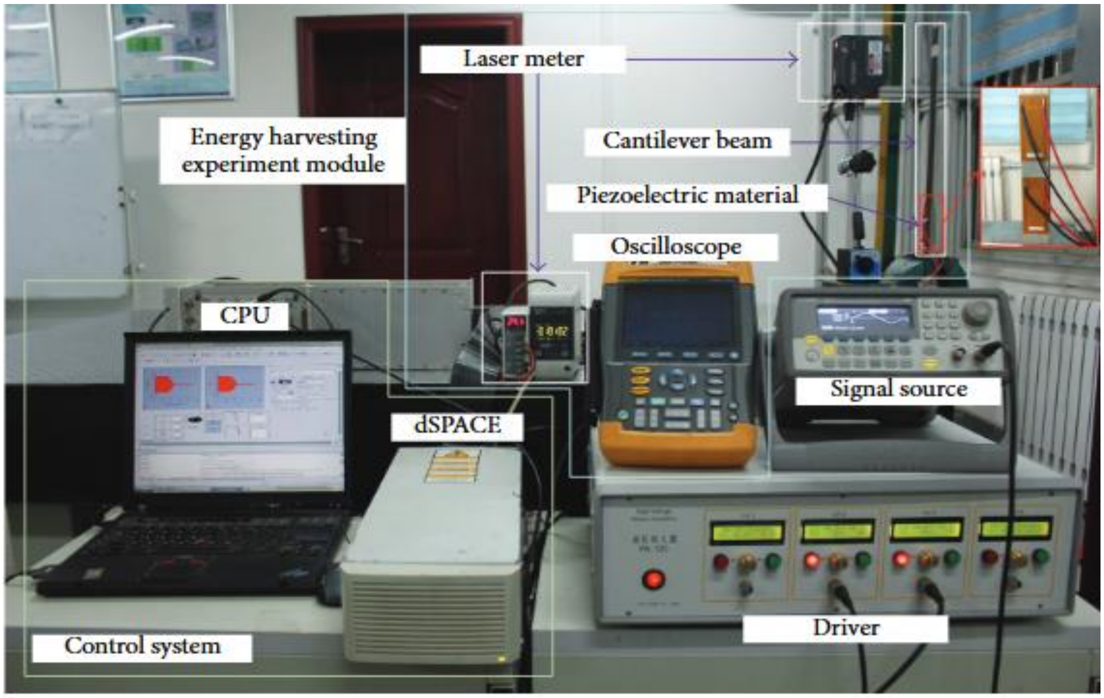

Fig. 1.1 Active control system with energy harvesting

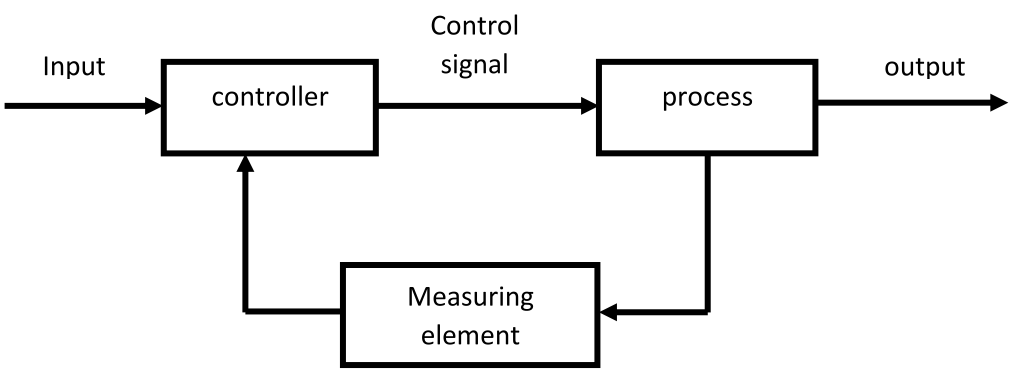

Fig. 1.2 closed loop-controlled system block diagram

The block diagram given in Fig. 1.2 illustrates in detail how to implement the closed loop control method from the engineering point of view. The control process can be summarized sequentially as follows:

1) The measuring element (Sensor) measures the instantaneous oscillations of the system as shown in Fig. 1.2
2) Using an analog-to-digital converter, the measured signal (output) is fed into a digital computer that works as a controller.