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قسم

نقسم باللّه العظيم أن المادة التي تم توثيقها وتسجيلها

علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات

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بعض الوثائق الأصلية تالفة
وبالرسالة صفحات لم ترد بالأصل

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TO

Spirit of my parents

My husband

My brother and my sisters

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SUMMARY

PREFACE

PREFACE

Associated with every mathematical programming, called the primal, there is another mathematical programming problem called its dual. These two problems posses very interseting and closely related properties. These properties were disscused in many paperes. In the following we shall concentrate on a few of these papers. Mangasarian in [11] (1969) introduced the duality for differentiable nonlinear programming problem and introduced the weak, the Wolfe's and the strict converse duality theorems. Osman, M. in [14] (1977) presented a qualitative analysis of basic notions such as the set of feasible parameters, the solvability set and the stability sets of the first and second kind for parametric convex programming problem with parameters in the constraints, also he introduced the dual problem of it, and defined the solvability set for the dual problem. And he proved that the solvability set of the primary problem equals the solvability set of its dual under some conditions. Schechter, M. in [24] (1979) extended the duality theorem of Wolfe [25] for nonlinear differential programming to the nondifferentiable case by replacing gradients by subgradients and he proved the converse duality theorem. Recently, a number of different duals for differentiable mathematical programs have been proposed by Mond and Weir in [12] (1981). The advantage of duals presented in [12] is that they allow the weakening of the usual convexity hypothesis needed for Wolfe duality [25] to hold. Osman, M. and el. in [16] (1986) introduced the same notions which were introduced in