Salwa Akl

بسم اللّه الرحمن الرحيم

مركز الشبكات وتكنولوجيا المعلومات

قسم التوثيق الإلكتروني

Salwa Akl

جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

قسم

نقسم باللّه العظيم أن المادة التي تم توثيقها وتسجيلها

علي هذه الأقراص المدمجة قد أعدت دون أية تغيرات

Salwa Akl

بعض الوثائق الأصلية تالفة
وبالرسالة صفحات لم ترد بالأصل

A STUDY OF SOME TYPES OF MAPPINGS ON TOPOLOGICAL SPACES

Thesis

SUBMITTED TO DEPARTMENT OF MATHEMATICS,
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IN

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By

ABD EL-FATTAH ABD EL-FATTAH YOSEIF EL-ATIK

Department of Mathematics,
Faculty of Science,
Tanta University.

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CURRICULUME VITAE

Name	:	Abd El-Fattah Abd El-Fattah Yosif El-Atik
Date of Birth	:	19/12/1970.
Locality	:	Ghrabia.
Nationality	:	Egyptian.
Qualifications	:	B. Sc. degree in (Mathematics) 1993 with grade excellent with Honours Faculty of Science, Tanta University.
Experience	:	Demonestrator at Departement of Mathematics, Faculty of Science, Tanta University.

Head of Departement of Mathematics

M. Beltagy

(Prof. Dr. M. Beltagy)

SUPERVISORS

Prof. Dr.

MOHAMMAD EZZAT ABD EL-MONSEF

Prof. of Pure Mathematics (Topology),
Vice Dean of Faculty of Science,
Tanta University

Dr.

FATMA ABDALLA HASSAN,
Lecturer of Pure Mathematics (Algebra),
Department of Mathematics,
Faculty of Science,
Tanta University

Dr.

IBRAHIM MOHAMMAD HANAFY,
Lecturer of Pure Mathematics (Topology),
Department of Mathematics
El-Arish, Faculty of Education,
Suez-Canal University

Head of Department of Mathematics

M. Beltagy

(Prof. Dr. M. Beltagy)

NOTE

Beside the work carried out in this thesis, the candidate Abd El-Fattah, A. El-Atik has attended some post-graduate courses for two academic years (93/94, 94/1995) in the following courses:

1- Algebra.

2- Numerical Analysis.

3- Partial Differential Equations.

4- Functional Analysis.

He has also successfully passed the final examinations in these topics in September 1995.

Head of Department of Mathematics

M. Beltagy

(Prof. Dr. M. Beltagy

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ABSTRACT

In this thesis, we introduce the concepts of γ-open and γ-closed sets and by using them we define the notions of γ-continuity, γ-open, γ-closed mappings and γ-homeomorphism. Some of their characterizations and connections between them with other corresponding ones are studied. Also, many γ-topological concepts as γ-closure, γ-interior, γ-exterior, ... etc and γ-density are investigated. we introduce and study the concept of γ-compact space. The connections between some types of mappings and γ-compact spaces are discussed. Finally, we define γ-connectedness depending on γ-separation notions. Several properties and connections between these notions and other existing ones are investigated.

CONTENTS

PREFACE

PREFACE

In the last thirty five years, mappings on spaces took a very important place in studying properties of spaces and in constructing new spaces from previously existing ones. So, several authors have been interested in this line of study and therefore many sorts of near continuous, near open and near closed mappings arose as one goes further in mathematics. Different types of mappings in topological spaces were introduced by many topologists. Firstly, in 1963, N. Levine [31] introduced the notions of semi-open sets and semi-continuity in a topological space and obtained a number of their properties. Finally, in 1983, M. E. Abd El-Monsef et.al [2] have presented the concepts of β-open sets and β-continuity. Throughout these years, O. Njasted [49] defined the concept of α-open sets which is weaker than openness. Also, A. S. Mashhour, et.al. [38] investigated another type of sets which is called preopen sets which is weaker than α-open sets. They also defined the concepts of precontinuous, preopen and preclosed mappings. All above notions generalized by A. S. Mashhour, et. al. [41,42] are called supraopen sets and supracontinuous mappings. Connections between them were constructed. Many authers [23, 24, 25, 27] have further extended the theory of topological spaces. For stimulating this branch of study, we use the union of preopen sets and semi-open sets to introduce a new topological notion called γ-open set

which is weaker than both of them. In general, this thesis consists of five chapters which are devoted to :

(i) Introduce and study the notions of γ-open and γ-closed sets.

(ii) Give and discuss the concepts of γ-continuous, γ-open, γ-closed and γ-irresolute mappings.

(iii) Construct the concepts of γ-compact spaces.

(iv) Introduce the concept of γ-connectedness by the use of γ-closure notion.

The introductory **Chapter "0"** gives an exposition of some needed definitions, preliminaries and conventions to be used throughout this thesis. Moreover some results concerning fundamental concepts in this work are introduced.

The purpose **in Chapter I** is to introduce and study new notions called γ-open (γ-closed) sets. The class of γ-open (γ-closed) sets contains the class of semi-open (semi-closed) sets as well as the class of preopen (preclosed) sets and contained in the class of β-open (β-closed) sets. Various notions like the γ-interior, γ-closure, γ-boundary, γ-exterior, γ-neighbourhood and the γ-derived set operators as well as set properties like γ-closedness are defined in § 1.1-1.3. Also, a weak form of density namely, γ-density is established in § 1.4. Many of their characterizations, properties and connections between them and other corresponding ones are also studied. Besides, effects of these new sorts on some topological spaces are studied.

weaker than connectedness [12, 58] is constructed. Several characterizations, properties and connections between these notions and other existing ones are investigated. Also, some algebraic properties of γ-connected product are presented and are successfuly jointed with topological invariant.

Many books such as [25, 67, 69] were valuable assistance in the preparation of this thesis.

Note (1) Some of new results in **Chapters I and II** are submitted for publication [4].

Note (2) The main results in **Chapter IV** have been presented for publication [5] in the proceeding of te 10th Topological Symposium, Jule 30-Aug. 1 (1996), El-Arish, Egypt.

CHAPTER 0

INTRODUCTION AND PRELIMINARIES

The first consideration in this chapter is to give an exposition of some needed definitions and useful preliminaries to be used throughout this thesis.

Throughout the present thesis (X,τ) and (Y,σ) are topological spaces or simply spaces with no separation axioms are assumed unless explicitly stated. For each $A \subset X$, the closure (resp. interior, boundary, exterior, derived) of A will be denoted by $\overline{A}$ (resp. A°, A^b, $ex(A)$, A^d). Also, $cl_\tau(A)(Int_\tau(A))$ denotes the closure (interior) of A with respect to τ in order to avoid confusion when there exist more than one topology on X. τ^c denotes the class of τ-closed subsets of X. $A^c = X - A$ means the complement of $A \subset X$, τ_A means the relative topology on $A \subset X$, the "neighbourhood" concept will be abbreviated "nbd" and N_x denotes the open nbd. system of a point $x \in X$ i.e. $N_x = \{U \in \tau : x \in U\}$. P(A) the power set of A is the family of all subsets of A. In the first article, the notions of nearly open and nearly closed sets are given and some relations between these concepts are discussed. While § 0.2 and § 0.3 are devoted to give a comperhensive survey on mappings which are near continuous, near open and near closed. We give the meaning of some known types of compact spaces, noumures of their topological properties are presented in the last article of this chapter.