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# بسم الله الرحمن الرحيم

مركز الشبكات وتكنولوجيا المعلومات

قسم التوثيق الإلكتروني



Salwa Ak1



# جامعة عين شمس

التوثيق الإلكتروني والميكروفيلم

## قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها

على هذه الأقراص المدمجة قد أعدت دون أية تغييرات



Salwa Akl



بعض الوثائق الأصلية تالفة  
وبالرسالة صفحات لم ترد بالأصل



**CELLULAR FOLDING OF SURFACES  
TO A POLYGON**

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**A THESIS**

SUBMITTED TO THE DEPARTMENT OF MATHEMATICS

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In

**(PURE MATHEMATICS)**

By

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# CELLULAR FOLDING OF SURFACES TO A POLYGON

BY

MOHAMMED RAMADAN EBRAHIM ZEEN EL-DEEN

## ABSTRACT

In this thesis we have developed the theory of certain continuous maps  $f : M \rightarrow P_n$  of such a surface  $M$  onto an  $n$ -sided polygon  $P_n$ . These maps are cellular folding. Then we obtained the relationship between the genus of  $M$  and the number  $n$ , either  $M$  is orientable or not.

For regular cellular folding of orientable surface of genus  $g > 1$  we proved that  $n \leq 2g + 2$ , and of nonorientable surface  $n \leq g + 2$ .



# INTRODUCTION

## INTRODUCTION

The category of CW-complexes is one of the most useful categories of topological spaces. CW-complexes have sufficiently nice local properties so that the most pathological situations are avoided, yet they are sufficiently general to include most spaces we need to consider in the thesis. These spaces are built up by successively attaching cells by arbitrary continuous maps.

The regular CW-complexes form a special category of CW-complexes which share with more general CW-complexes the advantage that often a space can be represented as a regular CW-complex with many fewer cells than in a simplicial decomposition.

Now, let  $M$  be a compact surface,  $f : M \rightarrow P_n$  be a continuous map onto an  $n$ -sided polygon  $P_n$ . These maps are called cellular, regular, foldings under certain conditions.

The aim of this thesis is to explore the relationship between the genus of  $M$  and the number  $n$ , either  $M$  is orientable or not.

The thesis consists of three chapters. The goal of the **first chapter** is to classify the compact connected topological 2-manifolds without boundary, surfaces. This to be done we needed to give first the idea about complexes, surfaces, connected sum, triangulations and orientation of manifolds.

**Chapter two** is dealing with regular CW-complexes. To discuss with these spaces we need first to give an idea about cell complexes, cell decomposition and CW-complexes. Also to be able to discuss the properties of cellular foldings we gave an idea about cellular maps and how to colour the graphs by either colouring the vertices or the edges.

In general a continuous map  $f: K \rightarrow L$  from a CW-complex  $K$  to another  $L$  is a cellular folding iff it maps  $i$ -cells of  $K$  to  $i$ -cells of  $L$  such that the closure of any cell  $e$  and its image  $f(e)$  must contains  $n$  distinct vertices.

**Chapter three** is presented for cellular, regular, folding over an  $n$ -sided polygon, consequently it is need first to discuss cellular, regular, foldings and its properties. Then we obtained the relationship between the genus of  $M$  and the number  $n$ . Also we proved that for a regular cellular folding of an orientable surface of genus  $g > 1$ , we have  $n \leq 2g + 2$  and for a nonorientable surface  $n \leq g + 2$ .