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التوثيق الإلكتروني والميكروفيلم

قسم

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On The Numerical Solutions For The Initial Value Problems In O.D.E. Using Spline Functions

A Thesis

Submitted By

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To the memory of Prof. Tharwat Fawzy,

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Summary

The main purpose of this work is to try to find a new technique for the solution of the third order initial value problem $y''' = F(x,y)$ under certain initial conditions using spline polynomials of degree 7 and 8.

The thesis consists of five chapters:

Chapter I is a general introduction in which we review some published results concerning the spline approximations for initial value problems.

In chapter II, an approximate procedure is introduced to solve the differential equation under consideration using spline polynomials of degree m, deficiency 4, i.e., $S(x) \in C^{m-4}[0,h]$, we prove the existence and uniqueness of the solution for this equation. Also we prove the stability for this method which is stable for $m=7,8$, unstable and hence divergent for $m \geq 9$.

In chapter III, we construct a spline polynomial of degree $7(m=7)$, that approximates the solution of the differential equation under consideration. We prove the consistency and convergence properties and show that the method is $o(h^{8-i})$ in $y^{(i)}(x)$ where $i=0(1)7$, that is:

$$\left|S^{(i)}(x) - y^{(i)}(x)\right| = o(h^{8-i}) \text{ for all } i=0(1)7.$$

In chapter IV, we use similar technique as that of chapter III to construct a spline polynomial of degree $m=8$. We also prove that the method is consistent. The convergence theorem in this case has the form:

$$\left|S^{(i)}(x) - y^{(i)}(x)\right| = o(h^{7-i}) \text{ for all } i=0(1)8.$$

In the last chapter, we give some numerical examples to illustrate the theoretical results obtained. The results are also presented graphically with the help of *mathemtica* software

CONTENTS

Chapter I

Introduction

The theory of spline functions and their applications, as stated by Schumaker [32] , is a relatively recent development. As late as 1960 there were no more than a handful of papers maintaining spline functions by name. Today more than thousand research papers have been published in this area. The rapid development of spline functions is due to their great usefulness in applications

The work presented in this thesis is about the numerical solution of Cauchy problems of ordinary differential equations. Many others have discussed the same problem (CF.[1-42] and the references therein).

In January 1967, Loscalzo and Talbot [10] had constructed an approximate solution to the differential equation :

$$y' = F(x,y) \quad , \quad y(0) = y_0 \tag{1.1}$$

which is a spline function of degree m ($m \geq 2$) and continuity class C^{m-1}. The function $F(x,y) \in C^{m-2}$ is assumed to satisfy Lipschitz condition with constant L in $T = \{(x,y) : 0 \leq x \leq b\}$. It had been found that the method is unstable for $m \geq 4$ and in the cases where $m = 2$ and 3 one has the trapazoidal rule and the Milne-Simpson method respectively.

In 1971, Micula [14] had found an approximate solution for the Cauchy problem for first order systems, which are polynomial splines of degree m ($m \geq 2$), continuity class C^{m-1}. It was shown that the method is unstable for $m \geq 4$.

Also, in 1971, Micula [15] extended his work to systems of n ordinary differential equations with initial values, while Loscalzo and Talbot [10] obtained results for the same problem in the one dimensional case.

In April 1983, Sallam and El-Hawary [29] investigated the existence, uniqueness and stability of spline vector valued function approximation of the Cauchy problem (1.1). The constructed spline function is a vector valued spline $S(x)$ of degree $m(m \geq 4)$ and continuity class C^{m-3} and the step size used is of length $3h$. This problem was studied earlier by Micula [15] who used spline function of full continuity (degree m and deficiency 1) and the step size was of length h.

Sallam and EL-Hawary [29] were able to show that their method is A-stable for $m = 4$, unstable and hence divergent for $m \geq 6$ and that it is stable for $m = 5$, while the method of Micula [15] was shown to be unstable and hence divergent for $m \geq 4$.

In June 1983, Sallam and El-Hawary [30] developed their work and discussed the convergence rate of the method in El-Hawary [29] and the question of consistency. They established the consistency for the spline of degree 4 and 5 respectively and hence the convergence. The advantage of their method over Micula [15] is that the method is $o(h^{m+1})$ and in class C^{m-3} for $m = 4$, 5.

In October 1983, Sallam and Hussien [31] investigated the existence, uniqueness and stability of spline vector valued function approximation of the second-order initial value problem:

$$y'' = F(x, y, y') \quad , \quad y(0) = y_0 \quad , \quad y'(0) = y'_0 \tag{1.2}$$

Assuming that the spline functions approximation $S(x)$ are of degree $m(m \geq 5)$, deficiency 3, i.e. $S(x) \subset C^{m-3}$ and that the step size is $H = 3h$, Sallam and Hussien [31] found that the approximate solution is stable for $m = 5$ and 6, unstable and hence divergent for $m \geq 7$, also the method is consistent for $m = 5$, 6 and the error bound is $o(h^{m+1})$.

In 1973, the same problem was also studied by Micula [18] who used a spline of order m, deficiency 1 and step size h and convergence was shown for $m \leq 4$.

In chapter III, we prove the convergence of the spline function for m=7 which approximates the solution of third order initial value problem , and show that the method is of order $o(h^{8-i})$ in $y^{(i)}(x)$, *i.e.*

$$\|S^{(i)}(x) - y^{(i)}(x)\| = o(h^{8-i}) \quad \text{for all } i = 0(1)7$$

also we prove its consistency.

In chapter IV, we prove the consistency for the spline function for $m = 8$ and also prove the convergence of spline approximations to the solution of the third order initial value problem, which has the form :

$$\|S^{(i)}(x) - y^{(i)}(x)\| = o(h^{9-i}) \quad \text{for all } i = 0(1)8$$

In the last chapter, a computer program with C-language has been written for calculating the coefficients of the spline polynomials of order 7 and 8 for solving the initial value problem of the third order and to calculate the error in that solution and its derivatives up to order m-4 for m=7,8. Then we introduce some numerical examples for testing the presented method. The obtained results for the function and its continuous derivatives are presented graphically using *mathemtaica* software.

Chapter II